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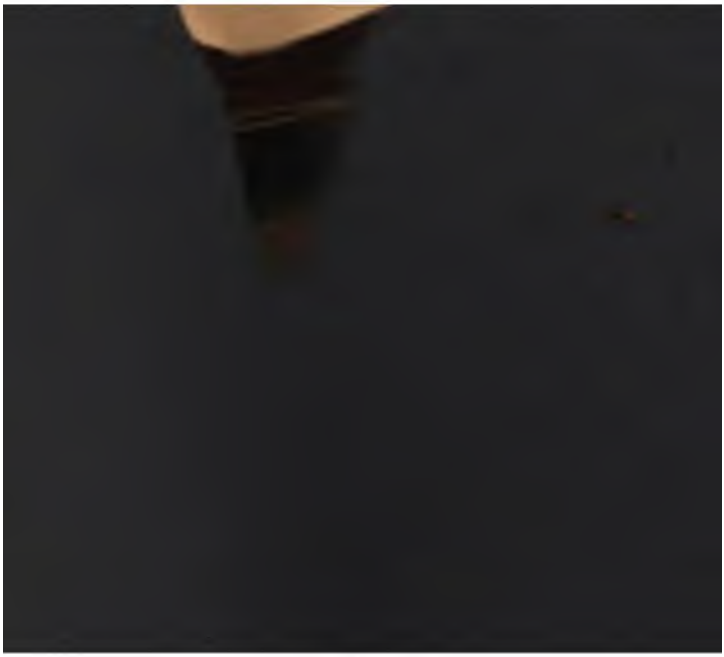
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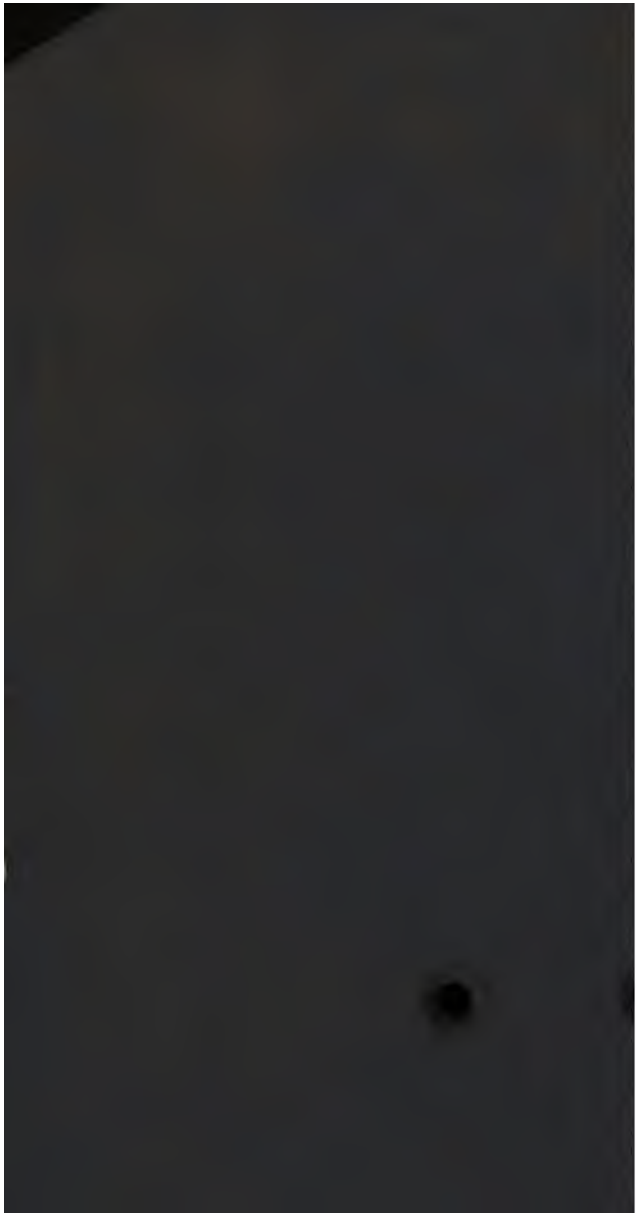
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EIGHTH EDITION:

Abridged for the Use of Schools, (under the Direction o from the Third Octavo Edition, *which was revised, improved, and more particularly adapted to the* Federa

BY NATHANIEL LORD, A. M

NEW-YORK:

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Sold by him, and by the Booksellers generally. - Al *ton*, by THOMAS & ANDREWS, Proprietors of the Co JANUARY, 1816.

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PREFACE

To the third octavo edition, from which this abridgment is made.

THE demand for this work still continuing, notwithstanding the publication of other works on Arithmetick and the higher branches of the Mathematicks, is evidence of its intrinsick merit, and has induced the proprietors of the copyright to present the publick with a new and improved edition.

Application was made to the Author, requesting him to revise and improve the work for a new edition; but he declined on account of want of health, and the Gentleman whom we employed, was engaged by the author's consent, and improved and corrected the work agreeably to his direction and advice.

The most important improvement in this edition, is the introduction of examples in the federal currency under each rule; and while this was considered necessary, in order to extend the knowledge and use of that currency, it was thought important not to omit examples in pounds, shillings and pence, which are, and will continue to be, the basis of many arithmetical questions; and therefore an acquaintance with them will always be useful.

Mr. NATHANIEL LORD, 3d, of Ipswich, the gentleman employed to correct and improve the work, has bestowed much attention upon it, and has received from Mr. PIKE all the information and advice he desired. The manner in which Mr. LORD has executed the task entrusted to him, will, we hope, gain additional reputation for the work, and entitle him to the thanks of the public. THOMAS & ANDREWS.

Boston, April, 1808.

EIGHTH EDITION.

In this edition all the errors discovered by the author, and respectable teachers, who make use of the work, have been carefully corrected—and much pains taken to have a good and correct edition.

January, 1816.

Explanation of the Characters used in this Treatise.

=	The sign of equality: as, 12 pence = 1 shilling, signifies that 12 pence are equal to one shilling; and, in general, that whatever precedes it, is equal to what follows.
+	The sign of addition: as, 5+5=10, that is, 5 added to 5 is equal to 10. Read 5 plus 5, or 5 more 5 equal to 10.
—	The sign of substraction: as, 12—4=8, that is, 12 lessened by 4, is equal to 8, or 4 from 12 and 8 remains. Read 12 minus 4, or 12 less 4 equal to 8.
×	The sign of multiplication: as, 6×5=30, that is, 6 multiplied by 5 is equal to 30. Read 6 into 5 equal to 30.
÷ or 6)30(5	The sign of division: as, 30÷5=6, that is, 30 divided by 5 is equal to 6. Read 30 by 5 equal to 6.
875 / 25	Numbers placed fraction-wise do also denote division, the numerator, or upper number being the dividend, and the denominator, or lower number, the divisor; thus, $\frac{875}{25}$ is the same as 875÷25=35.
∴ ∷ ∶	The sign of proportion, thus, 2 : 4 :: 8 : 16, that is, as 2 is to 4 so is 8 to 16.
∺	Signifies geometrical progression.
9—2+6=13	Shews that the difference between 2 and 9 added to 6 is equal to 13. Read 9 minus 2 plus 6 equal to 13. And that the line atop (called a *Vinculum*) connects all the numbers over which it is drawn.
12—3+5=4	Signifies that the sum of 3 and 5 taken from 12 leaves or is equal to 4.

$\overline{\ }^{2}$ Signifies the second power, or Square.

$\overline{\ }^{3}$ Signifies the third power, or Cube.

$\sqrt{\ }$ or $\overline{\ }^{\frac{1}{2}}$ Prefixed to any number or quantity, signifies that the square root of that number is required. It likewise (as also the character for any other root) stands for the expression of the root of that number or quantity to which it is prefixed.—As $\sqrt{36}=6$, and $\sqrt{\overline{108+36}}=12$, or $\overline{36}|^{\frac{1}{2}}=6$, &c.

$\sqrt[3]{\ }$ or $\overline{\ }^{\frac{1}{3}}$ Prefixed to any number, signifies that the cube root of that number is required or expressed. As $\sqrt[3]{216}=6$, and $\sqrt[3]{\overline{513+216}}=9$, &c.—or $\overline{216}|^{\frac{1}{3}}=6$, &c.

[The following should have been inserted in page 69, immediately before *Division of Vulgar Fractions*; and is now placed here, instead of its proper place, to keep this Edition page for page with the last.]

Another method for the Multiplication of mixed Quantities.

Case 1. To multiply a *whole number* by a *fraction*, or a *fraction* by a *whole number*.

RULE. Multiply the whole number by the numerator of the fraction, and divide the product by the denominator; But if the numerator be 1, divide by the denominator only.

	1.	2.	3.	4.	5.	6.	7.
Mult.	8	15	$\frac{1}{7}$	$\frac{1}{8}$	36	17	$\frac{3}{4}$
By	$\frac{1}{4}$	$\frac{1}{2}$	21	13	$\frac{2}{3}$	$\frac{3}{5}$	16
Prod.	2	$7\frac{1}{2}$	3	$2\frac{3}{8}$	3)72	5)51	4)48
				Prod.	24	$10\frac{1}{5}$	12

Case 2. To multiply a *whole number* by a *mixed one*.

RULE. Multiply by the fraction as in Case 1st; then multiply by the whole number, and add the two products, as in the examples.

	1.	2.	3.	4.	5.	6.
Mult.	15	35	68	42	129	$2\frac{3}{4}$
By	$3\frac{1}{2}$	$5\frac{1}{3}$	$7\frac{11}{12}$	$9\frac{3}{7}$	$8\frac{5}{6}$	11
	$7\frac{1}{2}$	$11\frac{2}{3}$	748	126	645	4)33
	45	175	$62\frac{4}{12}$	18	$80\frac{5}{6}$	$8\frac{1}{4}$
	$52\frac{1}{2}$	$186\frac{2}{3}$	476	378	1032	22
			$538\frac{4}{12}$	396	$1112\frac{5}{6}$	$30\frac{1}{4}$

Case 3. To multiply a *mixed number* by a *mixed number*.

RULE. Multiply the integral part of the multiplicand by the denominator of its fractional part, and add thereto its numerator: Then multiply by the mixed multiplier, by Case 2d, and divide the product by the denominator of the fractional part of the multiplicand, as in the following example.

Mult. $42\frac{3}{5}$
By $8\frac{2}{3}$

1st. $42\frac{3}{5}=213$ which mult. by $8\frac{2}{3}$

3)426
142
1704
5)1846

Product $= 369\frac{1}{5}$

After this manner may feet and inches be multiplied, calling 1 inch $\frac{1}{12}$ of a foot, 2 inches $\frac{1}{6}$, 3 inches $\frac{1}{4}$, 4 inches $\frac{1}{3}$, 5 inches $\frac{5}{12}$, 6 inches $\frac{1}{2}$, 7 inches $\frac{7}{12}$, 8 inches $\frac{2}{3}$, 9 inches $\frac{3}{4}$, 10 inches $\frac{5}{6}$, 11 inches $\frac{11}{12}$ of a foot.

ARITHMETICK.

ARITHMETICK is the Art or Science of computing by numbers, and is comprised under five principal or fundamental Rules, viz. *Notation* or *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

NUMERATION

Teaches the different values of figures by their different places, and to read or write any sum or number by these ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. 0 is called a cypher, and all the rest are called figures or digits.

2. Besides the simple values of figures as above noted, they have each a local value, according to the following law; viz. In a combination of figures, reckoning from right to left, the figure, in the first place, represents its primitive simple value; that in the second place, ten times its simple value, and so on; the value of the figure, in each succeeding place, being ten times the value of it, in that immediately preceding it.

3. The values of the places are estimated according to their order: the first is denominated the place of units; the second, tens; the third, hundreds; and so on, as in the Table. Thus, in the number 3467: 7, in the first place, signifies only seven; 6, in the second place, signifies 6 tens, or sixty; 4, in the third place, four hundred; 3, in the fourth place, three thousand; and the whole taken together, is read thus; three thousand four hundred and sixty-seven.

4. A cypher, though of no signification itself, yet possesses a place, and when set on the right hand of figures, in

whole numbers, increase their value in the same tenfold proportion; thus, 9 signifies only nine; but, if a cypher be placed on its right hand, thus, 90, it then becomes ninety.

To enumerate any parcel of figures, observe the following RULE.

First. Commit the words at the head of the Table, viz. units, tens, hundreds, &c. to memory; then, to the simple value of each figure, join the name of its place, beginning at the left hand and reading towards the right. *More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over such dot will, universally, have the name of thousands.—2. Place the figures, 1, 2, 3, 4, &c. as indices, over the 2d, 3d, 4th, &c. period: These indices will then show the number of times the millions are involved—the figure under 1 bearing the name of millions, that under 2, the name of billions, (or millions of millions) that under 3, trillions, (or millions of millions of millions.)

EXAMPLE.

Sextill.		Quintill.		Quatrill.		Trillions.		Billions.		Millions.		Units.	
th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	th	un	c.	x. t. c. x. u.
	6		5		4		3		2		1		
913,208	;	000,341	;	620,057	;	219,356	;	809,379	;	120,406	;	129,763	
	Thousands		Thousands		Thousands		Thousands		Thousands		Thousands		Thousands

NOTE 1. Billions is substituted for millions of millions; Trillions, for millions of millions of millions; Quatrillions, for millions of millions of millions of millions.

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. answer to millions so often involved as their indices respectively denote.

NOTE 2. The right hand figure of each half period has the place of units, of that half period; the middle one, that of tens, and the left hand one, that of hundreds.

To write Numbers.

Rule.—Write down the figures as their values are expressed, and supply any deficiency in their order with cyphers.

Application.

Write down, in proper figures, the following numbers.

Fifteen	15
Two hundred and seventy-nine	279
Three thousand four hundred and three	3403
Thirty seven thousand five hundred and sixty seven	37567
Four hundred and one thousand, and twenty eight	401028
Nine millions seventy two thousand and two hundred	9072200
Fifty five millions three hundred & nine thousand and nine	55309009
Eight hundred millions forty four thousand and fifty five	800044055

Write down, in words at length, the following numbers.

8	437	709040	3476194	7584397647
17	3010	379096	84094007	49163189186
129	76506	4091875	690748591	500098422700

Notation by Roman Letters.

I. One.	XL. Forty.
II. Two.	L. Fifty.
III. Three.	LX. Sixty.
IV. Four.	LXX. Seventy.
V. Five.	LXXX. Eighty.
VI. Six.	XC. Ninety.
VII. Seven.	C. One Hundred.
VIII. Eight.	CC. Two Hundred.
IX. Nine.	CCC. Three Hundred.
X. Ten.	CCCC. Four Hundred
XI. Eleven.	D. or IↃ. Five Hundred.
XII. Twelve.	DC. Six Hundred.
XIII. Thirteen.	DCC. Seven Hundred.
XIV. Fourteen.	DCCC. Eight Hundred.
XV. Fifteen.	DCCCC. Nine Hundred.
XVI. Sixteen.	M. or CIↃ. One Thousand.
XVII. Seventeen.	IↃↃ. Five Thousand.
XVIII. Eighteen.	IↃↃↃ. Fifty Thousand.
XIX. Nineteen.	IↃↃↃIↃↃↃ. Five Hundred Thous.
XX. Twenty.	MDCCCIX. One Thousand Eight
XXX. Thirty.	Hundred and Nine.

A less literal number, placed after a greater, always augments the value of the greater; if put before, it diminishes it. Thus, VI is 6: IV. is 4; XI is 11; IX is 9, &c.

ADDITION

IS the putting together of two or more numbers, or sums, to make them one total, or whole sum.

SIMPLE ADDITION

Is the adding of several integers, or whole numbers together, which are all of one kind, or sort; as, 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or sum total, is 39 pounds.

Rule.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units: After adding up every figure in that column, consider how many tens are contained in their sum, and placing the excess under the units, carry so many as you have tens, to the next column: Proceed in the same manner through every column or row, and set down the whole amount of the last row.

Proof.

Begin at the top of the sum, and reckon the figures downwards, in the same manner as they were added upwards, and, if it be right, this aggregate will be equal to the first. Or, cut off the upper line of figures, and find the amount of the rest; then if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

Addition and Subtraction Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column, and the other atop, and in the common angle of meeting, or at the right hand of the first, and under the second, you will find the sum—as, 5 and 8 are 13.

When you would subtract; find the number to be subtracted in the left hand column, run your eye along to the right hand till you find the number from which it is to be taken, and right over it, atop, you will find the difference—as 8 taken from 13 leaves 5.

Examples.

1.	2.	3.	4.	5.	6.
Units.	Tens. Units.	Hundreds. Tens. Units.	Thousands. Hundreds. Tens. Units.	X. Thous. Thousands. Hundreds. Tens. Units.	C. Thous. X. Thous. Thousands. Hundreds. Tens. Units.
9	34	374	7234	71614	116410
4	20	170	642	91740	94074
5	9	409	9	11947	7949
3	40	473	700	49	49490
7	10	108	6	10149	100946

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4566	2102865
4567890	4566	89064	4321234
5678209	333	654321	5682098
6789098	90	1234567	6546218

SUBTRACTION

Teaches to take a less number from a greater, to find a third, shewing the inequality, excess or difference between the given numbers; and it is both simple and compound.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of a like kind.

Rule.

Place the larger number uppermost, and the less underneath, so that units may stand under units, tens under tens, &c. then drawing a line underneath, begin with the units, and subtract the lower from the upper figure, and set down the remainder; but if the lower figure be greater than the upper, borrow ten and subtract the lower figure therefrom: To this difference add the upper figure, which being set down, you must add one to the next left hand place of the lower line, for that which you borrowed; and thus proceed through the whole.

Proof.

In either simple or compound Subtraction, add the remainder and the less line together, whose sum, if the work be right, will be equal to the greater line:—Or, subtract the remainder from the greater line, and the difference will be equal to the less.

EXAMPLES.

	1. £	2. £	3. Miles.	4. Yards.	5. Feet.	6. Cwt.
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
Rem.						
Proof						

	7.	8.	9.
	10020030040050060070080090	10000	1000000
	98076054032011023045067089	9999	1

MULTIPLICATION

MAY be accounted the most serviceable rule in Arithmetick. It teaches how to increase the greater of two numbers given, as often as there are units in the less; performs the work of many additions in the most compendious manner; brings numbers of great denominations into small, as pounds into shillings, pence, or farthings, &c.; and, by knowing the value of one thing, we find the value of many.

It consists of three parts.

1. The Multiplicand, or number given to be multiplied, and commonly the largest number.

2. The Multiplier, or number to multiply by, commonly the least number.

3. The Product is the result of the work, or the answer to the question.

SIMPLE MULTIPLICATION

Is the multiplying of any two numbers together, without having regard to their signification; as, 7 times 8 is 56, &c.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table for Multiplication;—Find your multiplier in the left hand column, and your multiplicand atop, and in the common angle of meeting, or against your multiplier along at the right hand, and under your multiplicand, you will find the product or answer.

To learn it for Division;—Find the Divisor in the left hand column, and run your eyes along the row to the right hand until you find the dividend: then directly over the dividend, atop, you will find the quotient, showing how often the divisor is contained in the dividend.

CASE I.

When the multiplier is not more than 12—Always placing the greatest number uppermost, set the multiplier underneath, units under units, &c. and begin as the Table directs, setting down the unit figure under units, and carrying the tens to the next place, in all respects as in simple addition.

PROOF.

Multiply the multiplier by the multiplicand.

EXAMPLES.

1.	2.	3.	7.
37934	769308	4980076	763896
2	3	4	5
Prod.			

5.	6.	7.	8.
67589	503764	3918295	9164785
6	7	8	9

9.	10.	11.
4879567	5864794	8583478646
10	11	12

CASE II.

When the multiplier is more than 12,—Multiply each figure in the multiplicand by every significant figure in the multiplier, beginning with the units, and placing the first figure of each product exactly under its multiplier: Lastly, add these several products together, in the same order, as they stand, and their sum will be their total product.

EXAMPLES.

	1.	2.	3.
Multiplicand	367	78041	786009
Multiplier	205	40097	37008
	1835		
	734		
Prod.	75235		

4.	5.	6.
647906	760483	91807584
4873	9152	6875
3157245938	6959940416	631589640000

CASE III.

When the multiplier is a composite number, that is, when it is *produced by the multiplication* of any two numbers in the

Table—Multiply the multiplicand by one of those figures first, and that product by the other; and the last product will be the total required.

EXAMPLES.

	1.	2.	3.
Multiply	59375 by 35	39187 by 48	91632 by 56
	7		
7×5=35			
	415625		
	5		
	2078125 Product.		

CASE IV.

When there are cyphers on the right hand of either the multiplicand, or multiplier, or both—Neglect those cyphers: then place the significant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand of the sum as many cyphers as there are at the right hand of both the factors.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380296000	306144000	8836301500000

CASE V.

To multiply by 10, 100, 1000, *&c.*—Set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.*

* This is evident from the nature of numbers, since every cypher annexed to the right of a number increases the value of that number a tenfold proportion.

EXAMPLES.

1.	2.	3.	4.
57935	84935	613975	8473065
10	100	1000	10000
Prod.			

DIVISION

TEACHES to separate any number, or quantity given, into any number of parts assigned; or to find how often one number is contained in another; or from any two numbers given, to find a third, which shall consist of so many units, as the one of those given numbers is comprehended in the other; and is a concise way of performing several subtractions.

There are four principal parts to be noticed in Division, viz.

1. The Dividend, or number given to be divided.
2. The Divisor, or number given to divide by.
3. The Quotient, or answer to the question, which shows how often the divisor is contained in the dividend.
4. The remainder (which is always less than the divisor, and of the same name with the dividend,) is very uncertain, as there is sometimes a remainder, and sometimes none.

Division is both simple and compound.

PROOF.

Multiply the divisor and quotient together, and add the remainder, if there be any, to the product: If the work be right, that sum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values; as, 56, divided by 8, produces 7 in the quotient: That is, 8 is contained 7 times in 56.

CASE I.

Rule.

First, seek how many times the divisor is contained in a competent number of the first figures of the dividend; when found, place the figure in the quotient; multiply the divisor by this quotient figure; place the product under the left hand figures of the dividend; then subtract it therefrom, and bring down the next figure of the dividend to the right hand of the remainder: If, when you have brought down a figure to the remainder, it is still less than the divisor, a cypher must be placed in the quotient, and another figure be brought down; after which you must seek, multiply, and subtract, till you have brought down every figure of the dividend.*

Examples.

Divisor. Divid. Quo.

3)175817(58605

15

25

24

18

18

17

15

2 Remainder.

Proof.

58605 Quotient.

×3 Divisor + 2

175817

* When there is no remainder to a division, the quotient is the absolute and perfect answer to the question; but where there is a remainder, it may be observed, that it goes so much towards another time as it approaches the divisor; thus, if the remainder be half the divisor, it will go half of a time more and so on; in order, therefore, to complete the quotient, put the last remainder to the end of it, above a line, and the divisor below it.

It is sometimes difficult to find how often the divisor may be had in the numbers of the several steps of the operation: The best way will be to find how often the first figure of the divisor may be had in the first, or two first figures of the dividend, and the answer made less by one or two, is generally the figure wanted; but if, after subtracting the product of the divisor and quotient from the dividend, *the remainder be equal* to, or exceed, the divisor, the quotient figure

In this example, I find that 3, the divisor, cannot be contained in the first figure of the dividend; therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend, &c. I then subtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz. 5; then, I inquire how often the divisor 3 is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when finding I cannot have the divisor in 1, I place 0 in the quotient, and bring down 7 to the one, and proceed as at the first.

Observe, that in multiplying by 3, I add in the 2.

2.

29)153598(5296
145

85
58

279
261

188
174

14

3.

6493)91876375(14150
6493

26946
25972

9743
6493

32507
32465

425

4.

28)503775(

5.

35)197184(

6.

85)994466(

7.

236)3798567(

8.

3479)483956795(

9.

5679)19647394(

10.

38473)119184693(

11.

641976)9187042959(

12.

5823789)791822376496(

must be increased accordingly; or if the product of the divisor and quotient figure exceed the dividend, then the quotient figure must be proportionally lessened.

13.

123456789)12193263111263526900(

CASE II.

When there is one cypher or more at the right hand of the divisor—It or they must be cut off; also cut off the same number of figures from the dividend, and then proceed as in case first: But the figures which were cut off from the dividend must be placed at the right hand of the remainder.

Examples.

1.

65 | 00)3794326 | 75(58374
325
544
520
243
195
482
455
276
260
1675 Remainder.

2.

5194 | 000)8937643 | 893(

3.

917 | 0)47658 | 3(

4.

875 | 000)91764789430 | 000(

5.

Quot. Rem.
1 | 0)9584 | 6

6.

Quot. Rem.
1 | 00)76495 | 80

7.

Quot. Rem.
1 | 000)93751839 | 462

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divisor, your work is done; those figures cut off at the right hand, are the remainder, and those on the left, the quotient, as above.

CASE III.

Short Division *is when the divisor does not exceed* 12.

Rule.

First, seek how often the divisor can be had in the first *figure, or figures* of the dividend; which, when found, place

in the quotient; then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left hand-figures of your dividend, and the units which remain must be accounted so many tens, which you must suppose to stand at the left hand of the next figure in the dividend, and to be reckoned with it; then, seek how often you can have your divisor in those two figures; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Divisor. Dividend.	2.	3.	4.
2)71935	3)51903	5)633795	6)8471937
Quo. 35967—1			
5	6	7	
8)5437846	9)54963784	11)91843756	

CASE IV.

When the divisor is such a number, that any two, or more, figures in the table, being multiplied together, will produce it, divide the given dividend by one of those figures; the quotient, thence arising, by the other, and so on: and the last quotient will be the answer.*

* As the learner, at present, is supposed to be unacquainted with the nature of Fractions, and as the quotient is incomplete without the remainder—I shall here give a rule for finding the remainder without having recourse to fractions.

RULE I.

Multiply the quotient by the divisor; subtract the product from the dividend, and the result will be the true remainder.

The Rule, which is most commonly made use of, when the divisor is a composite number, is,

RULE II.

Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on till you have gone through all the divisors and remainders, to the first.

EXAMPLES.

First Method.	Second Method
9)196473	8)196473
8)21830	9)24559
Quo. 2728—57	Quo. 2728—57

In the *first* operation, in dividing by 9, 3 remains, and by 8, 6 remains; which being the last remainder, I multiply it by the first divisor 9, and add in the first remainder 3, and they make 57, the true remainder. In the *second* method, dividing by 8, 1 remains, and by 9, 7 remains; I therefore multiply 7, the last remainder, by 8, adding in the 1, and they make 57, as before.

2.	3.	4.
25)167835	54)93738764.	132)38473682.

TABLES IN COMPOUND ADDITION.

1. FEDERAL MONEY.*

10 Mills	make 1	Cent	marked m. c.
10 Cents		Dime	d.
10 Dimes		Dollar	D.
10 Dollars		Eagle	E.

Mills.
10= 1 Cent
100= 10= 1 dime
1000= 100= 10= 1 dollar
10000=1000=100=10=1 eagle.

* It may be proper to introduce here an account of the Federal Money, as settled by Congress, the 8th of August, 1786, when it was *Resolved*,

"That the standard of the United States of America, for gold and silver, shall be eleven parts fine and one part alloy.

"That the Money Unit of the United States (being by the Resolve of Congress of the 6th of July, 1785, a Dollar) shall contain *of fine silver* $375\frac{64}{100}$ grains.

2. English Money.

4 Farthings	make 1	Penny	marked qrs. d.
12 Pence		Shilling	s.
20 Shillings		Pound	£ or l.

Farthings
4 = 1 Penny
48 = 12 = 1 Shilling
960 = 240 = 20 = 1 Pound.

"That the money of account, to correspond with the division of coins, agreeably to the above Resolve, proceed in a decimal ratio, agreeably to the forms and manner following, viz.

"Mill, the lowest money of account, of which 1000 shall be equal to the federal dollar, or money unit, 0·001

"Cent, the highest copper piece, of which 100 shall be equal to the federal dollar, 0·010

"Dime the lowest silver coin, of which 10 shall be equal to the dollar, 0·100

"Dollar, the highest silver coin, 1·000

"That betwixt the dollar and the lowest copper coin, as fixed by a resolve of Congress of the 6th of July, 1785, there shall be *three* silver coins, and *one* copper coin.

"That the silver coins shall be as follow: One coin containing $187\frac{82}{100}$ grains of fine silver, to be called *Half Dollar:* One coin containing $75\frac{128}{1000}$ grains of fine silver, to be called a *Double Dime:* and one coin containing $37\frac{564}{1000}$ grains of fine silver, to be called a *Dime.*

"That the two copper coins shall be as follow: One equal to the one hundredth part of the federal dollar, to be called a *Cent:* and one equal to the two hundredth part of the federal dollar, to be called a *Half Cent.*

"That $2\frac{1}{4}$ pounds Avoirdupois weight of copper shall constitute 100 Cents.

"That there shall be two gold coins: One containg $246\frac{268}{1000}$ grains of fine gold, equal to 10 dollars, to be stamped with the impression of the American Eagle, and to be called an *Eagle:* One containing $123\frac{134}{1000}$ grains of fine gold, equal to 5 dollars, to be stamped in like manner, and to be called a *Half Eagle.*

"That the mint price of one pound Troy weight of uncoined silver, eleven parts fine and one part alloy, shall be 9 dollars, 9 dimes, and 2 cents.

"That the mint price of one pound Troy weight of uncoined gold, eleven parts fine and one part alloy, shall be 209 dollars, 7 dimes, and 7 cents."

Pence Tables.

d.	s.	d.	d.	s.	d.
20	= 1	8	120	= 10	0
30	= 2	6	130	= 10	10
40	= 3	4	140	= 11	8
50	= 4	2	150	= 12	6
60	= 5	0	160	= 13	4
70	= 5	10	170	= 14	2
80	= 6	8	180	= 15	0
90	= 7	6	190	= 15	10
100	= 8	4	200	= 16	8
110	= 9	2	240	= 20	0

s.	d.	s.	d.
1	= 12	11	= 132
2	= 24	12	= 144
3	= 36	13	= 156
4	= 48	14	= 168
5	= 60	15	= 180
6	= 72	16	= 192
7	= 84	17	= 204
8	= 96	18	= 216
9	= 108	19	= 228
10	= 120	20	= 240

3. Troy Weight.*

			Marked
24 Grains	make 1	Pennyweight,	grs. pwt.
20 Pennyweights		Ounce	oz.
12 Ounces		Pound	℔ or lb.

4. Avoirdupois Weight.†

16 Drams	make 1 Ounce	marked	dr. oz.
16 Ounces	1 Pound		lb.
28 Pounds	¼ of a Hundred weight		qr.
4 Quarters	1 Hund. weight, or 112lbs.		Cwt.
20 Hundred weight	1 Ton.		T.

* By this weight are weighed Gold, Silver, Jewels, Electuaries, and all Liquors.

Note. 175 Troy ounces are precisely equal to 192 Avoirdupois ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1lb Troy=5760 grains, and 1lb Avoirdupois=7000 grains.

†By Avoirdupois are weighed all coarse and drossy goods, grocery and chandlery wares; bread, and all metals, except Gold and Silver.

5. Apothecaries Weight.*

20 Grains	make 1	Scruple	marked	gr. ℈
3 Scruples		Dram		ʒ
8 Drams		Ounce		℥
12 Ounces		Pound		lb.

6. Cloth Measure.

2¼ Inches	make 1	Nail	marked in. na.
4 Nails, or 9 Inches		Quarter of a Yard	qr.
4 Quarters, or 36 In.		Yard	yd.
3 Quarters, or 27 In.		Ell Flemish	E. Fl.
5 Quarters, or 45 In.		Ell English	E. E.
6 Quarters, or 54 In.		Ell French	E. Fr.

7. Long Measure.†

3 Barley corns	make 1	Inch	marked bar. in.
12 Inches		Foot	ft.
3 Feet		Yard	yd.
5½ Yards or 16½ ft.		Rod, Perch or Pole	pol.
40 Poles		Furlong	fur.
8 Furlongs		Mile	mile.
69½ Statute Miles, nearly		Degree of a great Circle	deg
360 Degrees		A great Circle of the Earth.	

* All the weights now used by Apothecaries, above grains, are Avoirdupois.

The Apothecaries' pound and ounce, and the pound and ounce Troy, are the same, only differently divided and subdivided.

† The use of Long Measure is to measure the distance of places or any other thing, where length is considered without regard to breadth.

Note 60 geometrical miles make a degree—4 inches a hand—5 feet a geometrical pace. 6 points make 1 line, 12 lines an inch, 12 inches a foot, and 6 feet one French toise, or fathom equal to 6 feet four inches 8·812875 lines English measure. 1 English foot equal to 11 inches 3·1154 lines French. 66 Feet, or 4 poles make a Gunters's chain. 3 miles make a league.

Or, in measuring Distances.

7$\frac{92}{100}$ Inches	make 1	Link
25 Links		Pole
100 Links		Chain
10 Chains		Furlong
8 Furlongs		Mile

8. TIME.*

60 Seconds	make 1	Minute	marked s. m.
60 Minutes		Hour	h.
24 Hours		Day	d.
7 Days		Week	w.
4 Weeks		Month	m.
13 Months 1 day & 6 hours		Julian Year	yr.

9. LAND, or SQUARE MEASURE.

144 Inches	make 1	Square Foot
9 Feet	——	Yard
30¼ Yards, or 272¼ Feet	——	Pole
40 Poles	——	Rood
4 Roods, or 160 Rods, or 4840 Yards	——	Acre
640 Acres	——	Mile

10. SOLID MEASURE.†

1728 Inches	make 1	Foot
27 Feet		Yard
40 Feet of round timber, or 50 Feet of hewn timber,		Ton or Load
128 Solid Feet, i. e. 8 in length, 4 in breadth, and 4 in height		Cord of Wood

*By the calender, the year is divided in the following manner: Thirty days hath September, April, June, and November; February twenty-eight alone, and all the rest have thirty-one.

When you can divide the year of our Lord by 4, without any remainder, it is then Bissextile, or Leap Year, in which February has 29 days.

†By Solid Measure are measured all things that have length, *breadth, and depth.*

11. Wine Measure.*

2 Pints	make 1	Quart	marked pt. qt.
4 Quarts		Gallon	gal.
10 Gallons		Anchor of Brandy	anc.
18 Gallons		Runlet	run.
31½ Gallons		Half Hogshead	½hhd.
42 Gallons		Tierce	tier.
63 Gallons		Hogshead	hhd.
2 Hogsheads		Pipe or Butt	P. or. B.
2 Pipes		Tun	Tun

12. Ale and Beer Measure.†

2 Pints	make 1	Quart	marked pt. qt.
4 Quarts		Gallon	gal.
8 Gallons		Firkin of Ale in London.	A. fir.
8½ Gallons		Firkin of Ale or Beer	
9 Gallons		Firkin of Beer in Lon.	B. fir.
2 Firkins		Kilderkin	kil.
2 Kilderkins		Barrel	Bar.
1½ Barrel, or 54 Gal.		Hogshead of Beer	hhd.
2 Barrels		Puncheon	pun.
3 Barrels, or 2 Hhds.		Butt.	butt.

13. Dry Measure.‡

2 Pints	make 1	Quart	marked pt. qt.
2 Quarts		Pottle	pot.
2 Pottles		Gallon	gal.
2 Gallons		Peck	pk.
4 Pecks		Bushel	Bu.
2 Bushels		Strike	str.
2 Strikes		Coom	co.
2 Cooms		Quarter	qr.
4 Quarters		Chaldron	ch.
4½ Quarters		Chaldron in London	
5 Quarters		Wey	wey.
2 Weys		Last	last.

* All Brandies, Spirits, Perry, Cider, Mead, Vinegar, Honey, and Oil, are measured by Wine Measure; Honey is commonly sold by the pound Avoirdupois.

† Milk is sold by the Beer quart.

‡ This measure is applied to all dry goods, as Corn, Seed, Fruits, Roots, Salt, Sand, Oysters, and Coal.

A Winchester bushel is 18½ inches diameter, and 8 inches deep.

COMPOUND ADDITION.

IS the adding of several numbers together, having different denominations, as, Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

Rule.

1. Place the numbers, so that those of the same denomination may stand directly under each other.

2. Add the right hand column or denomination together as in whole numbers; then divide the sum by as many of the same denomination, as make one of the next greater, setting down the remainder under the column added, and carry the quotient to the next superior denomination, continuing the same to the last, which add as in simple addition.

Examples.

1. Federal Money.

1.					2			3.		
E.	D.	d.	c.	m.	d.	c.	m.	d.	c.	m.
7	3	8	9	5	49	18	7	375		
	2	1	2	5	25	32	1	29	18	
9	0	0	5		93	7	5	7	12	5
	3	6	2	5	13	25	–	199	18	7
7	1	4	0	8		97	2	30	01	

2. English Money.

1.			2.				3.			
£	s.	d.	£	s.	d.	qrs.	£	s.	d.	qrs.
9	16	10	47	17	6	2	487	11	11	3
7	10	9	3	9	10	3	418	19	6	1
0	18	6	75	13	9	1	59	6	10	0
5	11	11	4	11	11	0	747	16	1	2

3. Troy Weight.

1.			2.				3.			
lb.	oz.	pwt.	lb.	oz.	pwt.	gr.	lb.	oz.	pwt.	gr
767	10	17	649	11	19	20	859	9	15	20
39	6	9	32	9	6	5	437	10	17	22
417	11	16	841	10	11	19	641	11	6	
935	9	17	473	9	17	23	738	9	12	18

4. Avoirdupois Weight.

1.			2.				3.					
lb.	oz.	dr.	T.	cwt.	qrs.	lb.	T.	cwt.	qrs.	lb.	oz.	dr
19	13	12	59	13	2	17	91	17	2	25	13	15
21	9	6	6	17	1	21	19	9	0	17	10	12
4	15	15	45	11	3	25	14	13	2	0	9	11
23	10	5	57	16	2	19	47	11	3	19	14	0

5. Apothecaries' Weight.

1.				2.				3.				
℥	℈	gr.		℥	℥	℈	gr.	lb.	℥	℥	℈	gr.
9	1	17		10	7	2	19	12	11	6	1	15
3	2	19		6	3	0	12	4	9	1	0	12
6	1	17		7	6	1	17	91	10	7	2	16
4	0	6		9	5	2	12	4	8	1	2	19

6. Cloth Measure.

1.			2.			3.		
Yd.	qr.	n.	E. E.	qr.	n.	E. Fl.	qr.	n.
76	2	3	91	3	2	75	2	1
3	3	1	49	4	3	7	1	3
42	3	3	6	2	3	84	0	2
57	2	2	84	4	1	76	2	3

7. Long Measure.

1.			2.			3						
Pol.	ft.	in.	Mil.	fur.	pol.	Deg.	m.	fu.	po.	ft.	in.	be.
12	11	10	9	7	36	759	56	6	29	15	10	2
9	10	9	7	3	19	317	39	1	36	11	6	1
8	12	11	4	1	24	497	63	7	24	9	8	1
7	15	6	6	5	12	562	17	0	11	13	11	0

8. Time.

1.					2.			3.						
W.	d.	h.	m.	s.	Y.	mo.	d.	Y.	mo.	w.	d.	h.	m.	s.
3	6	22	57	42	19	10	19	57	11	3	6	23	29	55
1	5	19	31	28	7	9	27	4	8	1	1	19	45	38
2	3	17	9	15	4	8	16	20	9	2	3	17	18	19
3	0	9	17	58	1	11	14	46	10	2	5	11	50	13

9. Land, or Square Measure.

1.			2.			3.				
Pol.	ft.	in.	Yd.	ft.	in.	Acr.	roo.	pol.	ft.	in.
36	179	137	28	7	119	756	3	37	245	128
19	248	119	9	3	75	29	1	28	93	25
12	96	75	29	6	120	416	3	31	128	119
18	110	122	4	8	12	37	1	19	218	20

10. Solid Measure.

1.			2.			3.		
Ton.	ft.	in.	Yd.	ft.	in.	Cord.	ft.	in.
29	36	1229	75	22	1412	37	119	1015
12	19	64	9	26	195	9	110	159
18	11	917	3	19	1091	48	127	1071
19	8	1001	28	15	1110	8	111	956

11. WINE MEASURE.

1.				2.				3.			
Tie.	gal.	qt.	pt.	Hhd.	gal.	qt.	pt.	Tun.	hhd.	gal.	qt.
37	39	3	1	51	53	1	1	37	2	37	2
9	17	2	1	27	39	3	0	19	1	59	1
4	28	0	0	9	18	0	1	28	2	0	0
32	19	1	1	0	9	2	1	19	0	47	1

12. ALE AND BEER MEASURE.

1			2.			3.		
A. B.	fir.	gal.	B. B.	fir.	gal.	Hhd.	gal.	qt.
49	3	7	29	1	8	379	58	3
26	2	3	19	3	5	19	0	1
9	0	4	16	0	3	121	37	2
17	3	0	9	1	8	467	19	1

13. DRY MEASURE.

1.				2.				3.			
Qr.	bu.	p.	qt.	Bu.	p.	qt.	pt.	Ch.	bu.	p.	qt.
64	7	3	7	37	2	5	1	37	27	3	5
9	4	1	5	19	3	7	0	6	29	1	7
19	6	2	1	16	2	0	1	15	30	0	0
4	0	2	0	5	1	6	1	4	11	3	0

COMPOUND SUBTRACTION.

Teaches to find the difference, inequality, or excess, between any two sums of divers denominations.

RULE.

Place those numbers under each other, which are of the same denomination, the less being below the greater, be-

gin with the least denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater; add the upper figure to them; from this sum subtract the lower, remembering always, to add one to the next superior denomination, for that which you borrowed.

Examples.

1. Federal Money.

	D.	c.	m.
From	39	15	5
Take	28	17	2

	E.	d.	c.	m.
	21	8	1	2
	10	7	5	

	D.	c.	m.
	100		
	48	87	5

	D.	c.
Borrowed	100	
Paid	20	18
Remains to pay		

	D.	c.	m.
Lent	200		
Received	145	50	
Due to me			

	D.	c.	m.
Borrowed	3000		
Paid	195		
at	1115	46	
several	247	37	5
times	995	12	5
Paid in all			
Remains to pay			

	D.	c.	m.
Lent	7159	12	8
Received	245	37	5
at	3112	15	7
several	2000		
times.	1092	92	0
Received in all			
Remains due			

2. English Money.

1.

	£.	s.	d.	qr.
Borrowed	349	15	6	1
Paid	195	11	8	1
Remains to pay	154	3	10	0

2.

	£.	s.	d.	qr.
Lent	701	9	8	1
Received	107	16	4	2
Due me				

3.

	£.	s.	d.	q.
rowed	19372	12	9	0
l at	293	16	8	0
	74	9	7	3
dry	9413	11	0	1
	1994	0	10	3
es.	3914	19	0	0
	1064	17	9	1
d in all				
n. to be paid				

4.

	£.	s.	d.	q.
Lent	27109	5	8	3
Received at several times.	5196	15	10	0
	384	16	6	2
	4187	18	11	1
	6649	16	8	0
	917	9	10	3
	3196	0	2	3
Rec. in all				
Rem. due				

3. Troy Weight.

	1. lb.	oz.	pwt.	gr.	2. lb.	oz.	pwt.	gr.	3. lb.	oz.	pwt.	gr.
ight	749	5	13	16	189	8	12	10	543	3	9	13
d	96	9	10	13	148	4	16	19	179	1	15	18
n.												

4. Avoirdupois Weight.

	1. lb.	oz.	dr.	2. C.	qr.	lb.	3. T.	cwt.	qr.	lb.	oz.	dr.
ight	7	9	12	8	2	13	9	11	3	17	5	12
d	3	12	9	4	1	15	3	12	1	19	10	9

5. Apothecaries Weight.

1. ℥	℥	ʒ	℈	gr.	2. lb.	℥	ʒ	℈	gr.	3. lb.	℥	ʒ	℈	gr.
9	8	1	1	13	65	10	6	2	1	84	1	1	1	1
8	4	1	1	16	31	8	4	2	0	65	9	3	1	17

6. Cloth Measure.

1.			2.			3.			4.		
Yd.	qr.	n.	E.E.	qr.	n.	E.F.	qr.	n.	E. Fr.	qr.	n.
35	1	2	467	3	1	765	1	3	549	4	2
19	1	3	291	3	2	149	2	1	197	4	3

7. Long Measure.

1.			2.			3.								
Yd.	ft.	in.	Mil.	fur.	pol.	Deg.	m.	fur.	p.	yd.	ft.	in.	bar.	
28	2	10	76	3	11	38	41	3	29	2	1	7	2	
17	2	11	27	3	21	19	35	5	31	3	1	9	1	

8. Time.

1.					2.			3.						
Mo.	d.	h.	m.	s.	Y.	mo.	d.	Y.	mo.	w.	d.	h.	m.	s.
6	17	13	27	19	7	3	13	48	9	2	4	19	27	31
1	21	16	41	35	4	2	19	19	9	3	5	20	19	39

9. Land or Square Measure

1.			2.			3.				
A.	R.	Pol.	A.	R.	Pol.	A.	R.	Pol.	ft.	in.
29	1	10	29	3	19	56	2	16	27	110
24	1	25	17	1	36	29	0	21	210	129

10. Solid Measure.

1.			2.			3.		
T.	ft.	in.	Yds.	ft.	in.	Cords.	ft.	in.
49	19	1100	79	11	917	349	97	1250
38	36	1296	17	25	1095	192	127	1849

11. Wine Measure.

1.				2.				3.		
Hhd.	gal.	qt.	pt.	Tier.	gal.	qt.		Tun.	hhd.	gal.
79	21	2	1	19	17	1		532	1	19
38	61	3	1	12	29	2		197	1	47

12. Ale and Beer Measure.

1.				2.					3.		
A.B.	fir.	gal.	qt.	B.B.	fir.	gal.	qt.	pt.	Hhd.	gall.	qt.
39	1	2	1	21	3	5	2	0	769	17	1
24	3	6	2	19	1	7	2	1	391	42	3

13. Dry Measure.

1.				2.				3.			
Qr.	bu	pk.	qt.	Bu.	pk.	qt.	pt.	Chal.	bu.	pk.	qt.
56	2	2	1	91	1	3	2	39	12	2	1
39	3	1	2	29	2	1	1	24	25	3	2

PROBLEMS

Resulting from a Comparison *of the preceding Rules.*

Prob. 1. Having the sum of two numbers, and one of them given to find the other.

Rule. Subtract the given number from the given sum, and the remainder will be the number required.

Let 288 be the sum of two numbers, one of which is 115, the other is required.

From 288 the sum,
Take 115 the given number

Remains 173 the other.

Prob. 2. Having the greater of two numbers, and the difference between that and the less, given to find the less.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other 198: What is the other?

From 325 the greater,
Take 198 the difference.

Rem. 127 the less.

PROB. 3. Having the least of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the less number,
198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule. To half the sum add half the difference, and the sum is the greater; and from half the sum take half the difference, and the remainder is the less.—Or, From the sum take the difference, and half the remainder is the least: To the least add the given difference, and the sum is the greatest.

What are those two numbers, whose sum is 48, and difference 14?

2)48 2)14

Half sum=24 Half diff. = 7

24+7=31 the greater; and 24—7=17 the less.

Or, 48—14÷2=17; and 17+14=31.

PROB. 5. Having the sum of two numbers and the difference of their squares* given, to find those numbers.

Rule. Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference: You will then have their sum and difference, to find the numbers by Problem 4.

What two numbers are those, whose sum is 32, and the difference of whose squares is 256?

* The square of a number is the product of it multiplied into *itself.*

Half sum 16
Half diff. 4

32)256(8 difference.
256

Greater 20

Less 12.

PROB. 6. Having the difference of two numbers, and the difference of their squares given, to find those numbers.

Rule. Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum; then proceed by Problem 4.

What are those two numbers, whose difference is 20, and the difference of whose squares is 2000?

20)2000(100 sum. 50+10=60, the greater; and
50—10=40 the less.

PROB. 7. Having the product of two numbers, and one of them given to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288, and one of them 8; I demand the other. 8)288

Ans. 36

PROB. 8. Having the dividend and quotient, to find the divisor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving division.

Given { 288 the dividend, / 36 the quotient.
Required the divisor

36)288(8 divisor.
288

PROB. 9. Having the divisor and quotient given, to find the dividend.

Rule. Multiply them together.

Given { 8 the divisor / 36 the quotient.
Required the dividend.

36
8

288 the dividend.

By a due consideration and application of these Problems only, many questions may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

APPLICATION OF THE PRECEDING RULES.

1. The least of two numbers is 19418, and the difference between them is 2384: What is the greater, and sum of both?

19418+2384=21802, greater; and 19481+21802=41220, sum.

2. Suppose a man born in the year 1743: When will he be 77 years of age? 1743+77=1820, Ans.

3. What number is that, which being added to 19418, will make 21802? 21802—19418=2384, Ans.

4. General Washington was born in 1732: What was his age in 1799? 1799—1732=67, Ans.

5. America was discovered by Columbus in 1492, and its independence was declared in 1776: How many years elapsed between those two eras?

1776—1492=284, Ans.

6. The massacre at Boston, by the British troops, happened March 5th, 1770, and the battle at Lexington, April 19th, 1775: How long between?

April 19, 1775—March 5, 1770=5y. 1m. 14d. Ans.

7. General Burgoyne and his army were captured Oct. 17th, 1777, and Earl Cornwallis and his army, Oct. 19th, 1781: What space of time between?

October 19, 1781—October 17, 1777=4y. 2d. Ans.

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783: How long did the war continue?

Jan. 20, 1783—April 19, 1775=7y. 9m. 1d.

9. L, M, N, and O purchased a quantity of goods in partnership; L paid £12 10s, a dollar,* and a crown† piece; M, 35s.; N, 29s. 10d.; and O, 79d.: What did the goods cost? Ans. £16 14 1.

10. A man borrowed, at different times, these several sums, viz. £29 5; £18 17 6; £45 12; £98, 3 dollars, one crown piece, and a half: How much was he in debt?

Ans. £193 2 6.

*6s. †6s. 8d.

11. There are 4 numbers: the first 317, the second 912, the third 1229, and the fourth as much as the other three, abating 97; What is the sum of all? Ans. 4819.

12. Bought a quantity of Goods for £125 10s. paid for truckage 45s. for freight 79s. 6d. for duties 35s. 10d. and my expenses were 53s. 9d.: What did the goods stand me in?
Ans, £136 4s. 1d.

13. A gentleman left his son £1725 more than his daughter, whose fortune was 15 thousand 15 hundred and 15 pounds: What was the son's portion, and what did the whole estate amount to?
Ans. The son's fortune, £18240, and the whole estate £34755.

14. A merchant had six debtors, who, together, owed him £2917 10s. 6d. A. B. C. D, and E, owed him £1675 13s. 9d. of it: What was F's debt? Ans. £1241 16s. 9d.

15. What is the difference between £1309 7s. 1d. and the amount of £345 13s. 4d. and £571 4s. 8d.?
Ans. £392 9s. 1d.

16. A merchant, at his first engaging in trade, owed £937 15s. he had in cash £1755 3s. 6d. in goods £459 12s. 3d. in good debts, £197 16s. and he cleared the first year £249 19s. 19d. What was the neat balance at the year's end? Ans. £1724 16s. 7d.

17. What sum of money must be divided between 12 men, so that each may receive £155?
£155×12=£1860 Ans.

18. What number must I multiply by 9, that the product may be 675? 675÷9=75 Ans.

19. A privateer of 175 men took a prize which amounted to £59 per man, besides the owner's half: What was the value of the prize? 175×59×2=£20650 Ans.

20. What is the difference between thrice five, and thirty, and thrice thirty five?
$\overline{35\times3}-\overline{5\times3+30}=60$, Ans.

21. The sum of two numbers is 750; the less 248: What is their difference, product, and the square of their difference?

750—248=502 the greater number, 502—248=254 difference, 502×248=124496 product, and 254×254=64516 square of the difference.

22. What is the difference between six dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the less?

Ans. $\overline{6\times12\times12}-\overline{6\times12}=792$ diff. $6\times12\times12\times6\times12=62208$ product, and $6\times12\times12\div\overline{6\times12}=12$, Quotient.

23. There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their sum and product are required.

Ans. $78\times25=1950$ the greater, $1950-\overline{15\times9}=1815$ the less. $1950+1815=3765$ the sum, and $1950\times1815=3539250$ the product.

24. A merchant began trade with £25327; for 6 years together, he cleared £1253 per annum; the next 5 years he cleared £1729 per annum; but, the last 4 years, had the misfortune to lose £3019 per annum: What was he worth at the 15 years end? Ans. £29414.

25. If a man spend £192 in a year, what is that per calender month; $192\div12=$£16. Ans.

26. If the Federal debt, which is 42,000,000 dollars, be equally divided between the thirteen states; what will be the share of each? Ans. 3230769$\frac{3}{13}$dols.

27. If 9000 men march in a column of 750 deep; how many march abreast? $9000\div750=12$, Ans.

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the same?

$\overline{3072\div32}-32=64$, Ans.

29. What number is that, which being multiplied by 3589, will produce 92050672?

$92050672\div3589=25648$, Ans.

30. Suppose the quotient arising from the division of two numbers to be 5879, and the divisor 37625: What is the dividend, if the remainder come out 9357?

$\overline{37625\times5379}+9357=202394232$, Ans.

REDUCTION

TEACHES to bring, or exchange numbers of one denomination to others of different denominations, retaining the same value.

It is of *two* sorts, viz. Descending and Ascending; the former of which is performed by multiplication, and the latter by Division.

REDUCTION DESCENDING.

Rule.

Multiply the highest denomination given, by so many of the next less, as make one of that greater, adding to the product parts of the same name; and thus continue, till you have brought it down as low as your question requires.

Proof.

Change the order of the question, and divide your last product by the last multiplier, and so on.

Note. From this rule and Case IV. of Simple Multiplication it appears, that Federal Money is reduced from higher to lower denominations by annexing as many cyphers as there are places from the denomination given to that required; or, if the given sum be of different denominations, by annexing the several figures of all the denominations in their order, and continuing with cyphers, if necessary, to the denomination required; or, what amounts to the same thing, by reading the whole number from the left to the required denomination, as one number in the required denomination.

Examples.

1. In 3 eagles 2 dollars, how many mills?
 Ans. 32000m.
2. In 91 dollars 75 cents, how many cents?
 Ans. 9175c.
3. In 50 eagles, how many dollars? Ans. 500D.
4. In 44 dollars 1 cent 4 mills, how many mills?
5. In 9 dollars 31 cents 7 mills, how many mills?
6. How many cents in 99 dollars 5 cents?
7. In 28 dollars 17 cents 5 mills, how many mills?

8. In £27 15s. 9d. 2qrs. how many farthings?

£	s.	d.	qrs.
27	15	9	2

Multiplied by 20 = shillings in a pound.

555 = shillings.

——— by 12 = pence in a shilling.

6669 = pence.

——— by 4 = farthings in a penny.

Ans. = 26678 farthings.

NOTE. In multiplying by 20, I added in the 15s.; by 12, the 9d.; and by 4, the 2qrs. which must always be done in like cases.

To prove the above question, change the order of it, and it will stand thus: In 26678 farthings, how many pounds?

4)26678 qr.

12)6669 2 qrs.

2|0)55|5 9d.

£27 15 9 2

9. In £36 12s. 10d. 1qr. how many farthings?
Ans. 35177.

10. In £95 11s. 5d. 3qrs. how many farthings?
Ans. 91751.

11. In £719 9s. 11d. how many half pence?
Ans. 345358.

12. In 29 guineas, at 28s. each, how many pence?
Ans. 9744.

13. In 37 pistoles, at 22s. how many shillings, pence and farthings? Ans. 814s. 9768d. 39072 farthings.

14. In 49 half johannes, at 48s. how many six pences?
Ans. 4704.

15. In 473 French crowns, at 6s. 8d. how many three pences? Ans. 12613$\frac{1}{3}$.

16. In 53 moidores, at 36s. how many shillings, pence and farthings? Ans. 1908s. 22896d. 91584qrs.

17. In £29 how many groats, threepences, pence and farthings?

Ans. 1740 groats, 2320 threepences, 6960d. 27840qrs.

18. Reduce 47 guineas and one fourth of a guinea into shillings, sixpences, groats, threepences, two pences, pence, and farthings.

Ans. 1323 shillings, 2646 sixpences, 3969 groats, 5292 threepences, 7938 twopences, 15876d. and 63504qrs.

REDUCTION ASCENDING.

Rule.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue, till you have brought it into that denomination which your question requires.

Note. From this rule and the note under Case II. of Simple Division, it appears, that Federal Money is reduced from lower to higher denominations by cutting off as many places as the given denomination stands to the right of that required; the figures cut off belonging to their respective denominations.

Examples.

1. How many eagles in 32000 mills? Ans. 3E. 2D.
2. In 9175 cents, how many dollars? Ans. 91D. 75c.
3. In 500 dollars, how many eagles? Ans. 50.
4. In 44014 mills, how many dimes?
5. In 9317 mills, how many dollars?
6. How many dollars in 28175 mills?
7. In 547325 farthings, how many pence, shillings, and pounds?

Farthings in a penny = 4)547325 qr.

Pence in a shilling = 12)136831d. 1qr.

Shillings in a pound = 2|0)11402|2s. 7d.

£570 2s. 7d. 1q.

Ans. 136831d.; 11402s.; 570£.

NOTE. The remainder is always of the same name as the dividend.

8. Bring 6177 farthings into pounds.

9. Bring 1751 farthings into pence, &c.

10. Bring 345358 half pence into pence, shillings and pounds.

11. Reduce 9744 pence into guineas, at 28s. per guinea.

12. In 39072 farthings, how many pistoles, at 22s ?

13. In 4704 six-pences, how many half-johannes ?

14. In 12613½ three-pences, how many French crowns, at 6s. 8d. ?

15. In 91584 farthings, how many moidores, at 36s, ?

16. In 27840 farthings, how many pence, three-pences, groats, shillings, and pounds ?

17. In 63504 farthings, how many pence, two-pences, three-pences, groats, six-pences, shillings, and guineas ?

NOTE. The preceding questions may serve as proofs to those in Reduction descending.

REDUCTION DESCENDING and ASCENDING.

1. MONEY.

1. In £97 how many pence, and English or French crowns, at 6s. 8d. ? Ans. 23280d. and 291 crowns.

2. In 519 English half crowns, how many pence and pounds ? Ans. 20760d. and 86l. 10s.

3. In 735 French crowns, how many shillings and French guineas, at 26s. 8d. ? Ans. 4900s. and 183 guineas, 20s.

4. In 5793 pence, how many farthings, pounds and pistoles ?

Ans. 23172qrs. ; £24. 2s. 9d.; and 21 pistoles, 20s. 9d.

5. In 59 half joes, 37 moidores, 45 guineas, 63 pistoles, 24 English crowns, and 19 dollars, how many pounds,

half joes, moidores, guineas, pistoles, English crowns, dollars, shillings, pence and farthings?

Ans. 354l. 4s.; 147 half joes, 28s.; 196 moidores, 28s.; 253 guineas; 322 pistoles; 1062 English crowns, 4s.; 1180 dollars, 4s.; 7084 shillings; 85008 pence, and 340032 farthings.

When it is required to know how many sorts of coin, of different values, and of equal number, are contained in any number of another kind; reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor; then reduce the money given into the same denomination for a dividend, and the quotient arising from the division will be the number required.

NOTE. Observe the same direction in weights and measures.

1. In 275 half Johannas, how many moidores, guineas, pistoles, dollars, shillings, and six-pences, of each the like number?

A moid. is 36s.	= 72 sixpences	275 half joes.
A guinea is 28s.	= 56 do.	48s. in a johan.
A pistole is 22s.	= 44 do.	
A dollar is 6s.	= 12 do.	2200
One shilling	= 2 do.	1100
	1 do.	
		13200 shillings.
Divisor	= 187 sixpences.	2 six-p. in a shil.
		Dividend = 26400 sixpences.

187)26400(141 of each, and 33 sixpences, or 16s. 6d. over, the answer.

2. A gentleman distributed £37. 10s. between 4 persons in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third 10s. and the fourth 5s. What did each person receive?

Ans. The first man £15.; second £11. 5s.; third £7. 10s.; fourth £3. 15s.

2. TROY WEIGHT.

1. How many grains in a silver bowl, that weighs 3lb. 10 oz. 12 pwt.?

lb. oz. pwt.
3 10 12
12 ounces in a pound.

46 ounces
20 pennyweights in an ounce.

932 pennyweights.
24 grains in one pwt.

3728
1864

Proof 24)22368 grains, Ans.
2 | 0)93|2

12)46 12 pwt.

3lb. 10 oz.

2. In 13 ingots of gold, each weighing 9oz. 5pwt. how many grains? Ans. 57720gr.

3. In 97397 grains, how many pounds?
Ans. 16lb. 10oz. 18pwt. 5gr.

4. How many rings, each weighing 5pwt. 7gr. may be made of 3lb. 5oz. 16pwt. 2gr. of gold? Ans. 158.

3. Avoirdupois Weight.

Cwt. qrs. lb. oz.
1. In 91 3 17 14 how many ounces?
4

367 quarters.
28

2943
735

10293 pounds.
16

61762
10293

164702 ounces.

Proof.
16)164702

28)10293 14oz.

4)367 17lb.

Cwt. 91 3qrs.

2. In 12 tons, 15cwt. 1qr. 19lb. 6oz. 12dr. how many drams? Ans. 7323500dr.

3. In 24lb. 11oz. 9dr. how many drams? Ans. 6329dr.

4. In 44800 pounds, how many drams and tons? Ans. 11468800dr. and 20 tons.

5. In 28 pounds avoirdupois, how many pounds Troy?

28lb.
7000 grains in 1lb. avoir.

grs. in 1lb. Tr.=576|0)196000(340lb.
1728

2320
2304

160
12

576|0)1920(0 oz.
20

576|0)3840(6pwt.
3456

3840
24

1536
768

576|0)9216(16gr.
576

3456
3456

6. In 47lb. 9oz. 13pwt. 17grs. Troy, how many pound avoirdupois?

lb. oz. pwt. gr.
47 9 13 17
12

573
20

11473
24

45899
22947

7|000)275|369(39lb.
21

65
63

2369
16

14214
2369

7|000)37|904(5oz.
35

2904
16

17424
2904

7|000)46|464(6$\frac{4464}{7000}$ dr.
42

4464

4. Apothecaries Weight.

1. How many grains are there in 37lb. 6ℨ ?

lb.	ℨ
37	6
12	
450	ounces.
8	
3600	drams.
3	
10800	scruples.
20	
216000	grains.

Proof.

2|0)21600|0

3)10800

8)3600

12)450

37lb. 6ℨ

2. In 9lb. 8ℨ 1ℨ 2℈ 19gr. how many grains ?

Ans. 55799gr.

3. In 55799 grains, how many pounds, &c. ?

Ans. 9lb. 8oz. 1ℨ 2℈ 19gr.

5. Cloth Measure.

1. In 127 yards, how many quarters and nails ?

Yds.	
127	
4	
508	qrs.
4	

Ans. 2032 nails.

Proof.

4)2032

4)508

127 yds.

2. In 9173 nails, how many yards ?

Ans. 573yds. 1qr. 1n.

3. In 75 ells English, how many quarters and nails ?

Ans. 375qrs. 1500n.

4. In 56 ells Flemish, how many quarters and nails ?

Ans. 168qrs. 672n.

5. In 7248 nails, how many yards, ells Flemish, and ells English ?

Ans. 453yds. 604 ells Flemish, 362 ells Eng. 2qrs.

6. In 19 pieces of cloth, each 15yds. 2qrs. how many yards, quarters and nails?

Ans. 294yds. 2qrs.; 1178qrs. and 4712n.

6. Long Measure.

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles?

Miles.	Proof.
43	3)8173440
8	
344 furlongs.	12)2724480
40	
13760 rods.	3)227040
5½	
68800	11)75680
6880	
75680 yards.	6880
3	2
227040 feet	4\|0)1376\|0
12	
2724480 inches.	8)344
3	
8173440 Ans.	43

Here I divide by 11 and multiply the quotient by 2, because twice 5½ is 11;—or, I might first have multiplied by 2, and then have divided the product by 11.

2. How many barley-corns will reach round the globe, it being 360 degrees? Ans. 4755801600

3. How many inches from Newburyport to London, it being 2700 miles? Ans. 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles? Ans. 13440 times.

5. In 190080 inches, how many yards and leagues?
Ans. 5280yds. and 1 league.

7. Time.

1. In 20 years, how many seconds?

d.	h.	Proof.
365	6 in a year.	6\|0)631152000
24		6\|0)10519200
1466		2\|0)17532\|0
730		4×6)8766
8766	hours in one year.	4)1461
20		365d. 6h.
175320	hours in 20 years.	
60		
10519200	minutes in do.	
60		
631152000	seconds in do.	

2. Suppose your age to be 15y. 19d. 11h. 37m. 45s. how many seconds are there in it, allowing 365 days 6 hours to the year? Ans. 475047465.

3. How many minutes from the first day of January to the 14th day of August, inclusively? Ans. 325440.

4. How many days since the commencement of the Christian era?

5. How many minutes since the commencement of the American war, which happened on the 19th day of April, 1775?

6. How many seconds between the commencement of the war, April 19th, 1775, and the Independence of the United States of America, which took place the 4th day of July, 1776?* Ans. 38188800.

* 1776 was a leap year.

8. Land or Square Measure.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

Acr.	r.	p.
29	3	19
4		
119	roods.	
40		

Ans. 4779 perches.

Proof.

4|0)477|9

4)119 19p.

29a. 3r.

2. In 1997 poles, how many acres? Ans. 12a. 1r. 37p.

3. In 89763 square yards, how many acres, &c.?
Ans. 18a. 2r. 7p. 101ft. 36in.

9. Solid Measure.

1. In 15 tons of hewn timber, how many solid inches?

15	tons.
50	
750	feet.
1728	
6000	
1500	
5250	
750	

1296000 inches, Ans.

Proof.

5|0

1728)1296000(75|0

12096

8640

8640

15 To.

2. In 9 tons of round timber, how many inches?
Ans. 622080.

3. In 25 cords of wood, how many inches?
Ans. 5529600.

10. Wine Measure.

1. In 9hhd. 15gal. 3qt. of wine, how many quarts?

hhd.	gal.	qt.
9	15	3
63		
32		
55		
582	gallons.	
4		
2331	quarts, Ans.	

Proof.

4)2331

63)582 3qts.

9hhd. 15gal.

2. In 12 pipes of wine, how many pints?

Ans. 12096.

3. In 9758 pints of brandy, how many pipes?

Ans. 9p. 1hhd. 22gal. 3qts.

11. Ale and Beer Measure.

1. In 29hhd. of beer, how many pints?

hhd.		
29		
54		
116		
145		
1566	gallons.	
4		
6264	quarts.	
2		
12528	pints.	

Proof.

2)12528

4)6264

54)1566

29hhd.

2. In 47 barrels, 18gal. of ale, how many pints?

Ans. 13680.

3. In 36 puncheons of beer, how many butts?

Ans. 24.

12. Dry Measure.

1. In 42 chaldrons of coals, how many pecks?

Chaldrons.

42	Proof.
32	4)5376
84	32)1344(42
126	128
1344 bushels.	64
4	64

Ans. 5376 pecks.

2. In 75 bushels of corn, how many pints? Ans. 4800.

3. In 9376 quarts, how many bushels? Ans. 293.

VULGAR FRACTIONS.

FRACTIONS, or broken numbers, are expressions for any assignable parts of a unit, or whole number; and are represented by two numbers, placed one above another, with a line drawn between them, thus, $\frac{5}{8}$, $\frac{4}{3}$, &c. signifying five eighths, four thirds, that is, one and one third, &c.

The figure above the line is called the numerator, and that below, the denominator.

The denominator (which is the divisor in division) shows how many parts the integer is divided into; and the numerator (which is the remainder after division) shows how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed. Any whole number may be made an improper fraction, by drawing a line under it, and putting unity, or 1, for a denominator, as 9 may be expressed fractionwise, thus $\frac{9}{1}$, and 12 thus, $\frac{12}{1}$, &c.

1. A *single* or *simple* fraction is a fraction expressed in a simple form; as, $\frac{1}{2}$, $\frac{5}{9}$, $\frac{7}{16}$, &c.

2. A *compound* fraction is a fraction expressed in a compound form, being a fraction of a fraction; or two or more fractions connected together; as, $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{7}$ of $\frac{5}{11}$ of $\frac{11}{13}$; which are read thus, one half of three fourths, two sevenths of five elevenths of nineteen twentieths, &c.

3. A *proper* fraction, is a fraction whose numerator is less than its denominator; as, $\frac{2}{3}$, $\frac{3}{4}$, &c.

4. An *improper* fraction, is a fraction whose numerator exceeds its denominator; as $\frac{5}{2}$, $\frac{8}{3}$, &c.

5. A *mixed number* is composed of a whole number and a fraction; as, $7\frac{3}{4}$, $35\frac{4}{13}$, &c. that is, seven and three fifths, &c.

6. A fraction is said to be in its *least* or *lowest terms*, when it is expressed by the least numbers possible.

7. The *common measure* of two, or more numbers, is that number, which will divide each of them without a remainder: Thus, 5 is the common measure of 10, 20, and 30; and the *greatest* number which will do this, is called the *greatest common measure.*

8. A number, which can be measured by two, or more, numbers, is called their *common multiple:* And, if it be the least number, which can be so measured, it is called the *least common multiple;* thus, 40, 60, 80, 100, are multiples of 4 and 5; but their least common multiple is 20.

9. A *prime number*, is that which can only be measured by itself, or an unit.

10. That number which is produced by multiplying several numbers together, is called a *composite number.*

11. A *perfect number* is equal to the sum of all its aliquot parts.

Problem I.

To find the greatest common measure of two or more numbers.

Rule.

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain, then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them, as before; then

of that common measure and one of the other numbers, and so on, through all the numbers to the last; then will the greatest common measure, last found, be the answer.

3. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.

Examples.

1. What is the greatest common measure of 1836, 3996, and 1044?

1836)3996(2
3672
324)1836(5
1620
216)324(1
216
Com. meas. =108)216(2
216

So 108 is the greatest common measure of 3996 and 1836;
Hence 108)1044(9
972
72)108(1
72
Last gr. com. meas.=36)72(2
72

Therefore 36 is the answer required.

2. What is the greatest common measure of 1224 and 1080? Ans. 72.

Problem II.

To find the least common multiple of two, or more, numbers.

1. Divide by any number that will divide two or more of the given numbers, without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2. Divide the second line as before, and so on, till there are no two numbers that can be divided; then, the continued product of the divisors and quotients will give the multiple required.

Examples.

1. What is the least common multiple of 6, 10, 16, and 20?

*5)	6	10	16	20
*2)	6	2	16	4
*2)	3	1	8	2
	*3	1	*4	1

$5\times2\times2\times3\times4=240$ Ans.

I survey my given numbers, and find that 5 will divide two of them, viz. 10 and 20, which I divide by 5, bringing into a line with the quotients, the numbers which 5 will not measure: again, I view the numbers in the second line, and find two will measure them all, and I get 3, 1, 8, 2, in the third line, and find that two will measure 8 and 2, and in the fourth line get 3, 1, 4, 1; all prime; I then multiply the prime numbers and the divisors continually into each other, for the number sought, and find it to be 240.

2. What is the least common multiple of 6 and 8?
Ans. 24.

3. What is the least number that 3, 5, 8, and 10 will measure? Ans. 120.

4. What is the least number which can be divided by the 9 digits separately, without a remainder? Ans. 2520.

REDUCTION OF VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.

To abbreviate, or reduce fractions to their lowest terms.

RULE.*

Divide the terms of the given fraction by any number which will divide them without a remainder, and the quo-

* That dividing both the terms, that is, both numerator and denominator of the fractions, equally by any number, whatever, will give another fraction, equal to the former, is evident: And if those

tients again in the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms.

Or, Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

Examples.

1. Reduce $\frac{144}{180}$ to its lowest terms.

$$8\left\{\frac{144}{180}=\frac{36}{45}=\frac{12}{15}=\frac{4}{5}\right.$$ the answer.

(4) (3)

Or thus:

288)480(1
288

192)288(1
192

Com. mea. 96)192(2
192

Therefore 96 is the greatest common measure:

and $96\left\{\frac{144}{180}=\frac{3}{4}\right.$ the same as before.

2. Reduce $\frac{77}{121}$ to its lowest terms. Ans. $\frac{7}{11}$.

3. Reduce $\frac{44}{176}$ to its lowest terms. Ans. $\frac{1}{4}$.

4. Reduce $\frac{1111}{3333}$ to its lowest terms. Ans. $\frac{1}{3}$.

divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. When numbers with the sign of addition or subtraction between them, are to be divided by any numbers, each of the numbers must be divided: Thus, $6+9+12=2+3+4=9$.

3

2. But if the numbers have the sign of multiplication between them; then only one of them must be divided: Thus,

$$\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24.$$

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

36
×8+5
——
Ans. 293
——
8

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum, 293, is the numerator of the fraction sought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$

Or, $\frac{36\times8+5}{8}=\frac{293}{8}$ Ans. as before.

2. Reduce $127\frac{4}{7}$ to its equivalent improper fraction.
Ans. $\frac{893}{7}$.

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.
Ans. $\frac{12410}{19}$.

CASE III.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8. $6\times8=48$, and $\frac{48}{8}$ the Ans.—Proof. $\frac{48}{8}=48\div8=6$.

2. Reduce 15 to a fraction, whose denominator shall be 12.
Ans. $\frac{180}{12}$.

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$.

tients again in the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms.

Or, Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

Examples.

1. Reduce $\frac{144}{192}$ to its lowest terms.

$8\left\{\frac{144}{192}=\frac{(4)\ 18}{(4)\ 24}=\frac{(3)\ 6}{(3)\ 8}=\frac{3}{4}\right.$ the answer.

Or thus:

288)480(1
288

192)288(1
192

Com. mea. 96)192(2
192

Therefore 96 is the greatest common measure:

and $96\left\{\frac{144}{192}=\frac{3}{4}\right.$ the same as before.

2. Reduce $\frac{17}{68}$ to its lowest terms. Ans. $\frac{1}{4}$.

3. Reduce $\frac{44}{176}$ to its lowest terms. Ans. $\frac{1}{4}$.

4. Reduce $\frac{1111}{3333}$ to its lowest terms. Ans. $\frac{1}{3}$.

divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. When numbers with the sign of addition or subtraction between them, are to be divided by any numbers, each of the numbers must be divided: Thus, $\frac{6+9+12}{3}=2+3+4=9$.

2. But if the numbers have the sign of multiplication between them; then only one of them must be divided: Thus, $\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

36	I multiply 36 by 8, and adding the
$\times 8+5$	numerator 5 to the product, as I multiply,
Ans. 293	the sum, 293, is the numerator of the
8	fraction sought, and 8 the denominator:
	So that $\frac{293}{8}$ is the improper fraction,
	equal to $36\frac{5}{8}$

Or, $\frac{36\times 8+5}{8}=\frac{293}{8}$ Ans. as before.

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction.
Ans. $\frac{2163}{17}$.

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.
Ans. $\frac{12410}{19}$.

CASE III.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8. $6\times 8=48$, and $\frac{48}{8}$ the Ans.—Proof. $\frac{48}{8}=48\div 8=6$.

2. Reduce 15 to a fraction, whose denominator shall be 12.
Ans. $\frac{180}{12}$.

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$.

tients again in the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms.

Or, Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

Examples.

1. Reduce $\frac{144}{192}$ to its lowest terms.

$$8\left\{\frac{144}{192}=\frac{18}{24}=\frac{9}{12}=\frac{3}{4}\right. \text{ the answer.}$$

(4) (3)

Or thus:

288)480(1
288
192)288(1
192
Com.mea. 96)192(2
192

Therefore 96 is the greatest common measure:

and $96\left\{\frac{144}{192}=\frac{3}{4}\right.$ the same as before.

2. Reduce $\frac{17}{170}$ to its lowest terms. Ans. $\frac{1}{10}$.
3. Reduce $\frac{44}{176}$ to its lowest terms. Ans. $\frac{1}{4}$.
4. Reduce $\frac{1111}{1111}$ to its lowest terms. Ans. $\frac{1}{1}$.

divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. When numbers with the sign of addition or subtraction between them, are to be divided by any numbers, each of the numbers must be divided: Thus, $\frac{6+9+12}{3}=2+3+4=9$.

2. But if the numbers have the sign of multiplication between them; then only one of them must be divided: Thus, $\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

36 ×8+5 ——— Ans. 293 ——— 8

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum, 293, is the numerator of the fraction sought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$

Or, $\frac{36\times8+5}{8}=\frac{293}{8}$ Ans. as before.

2. Reduce $127\frac{4}{7}$ to its equivalent improper fraction.
Ans. $\frac{893}{7}$.

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.
Ans. $\frac{12410}{19}$.

CASE III.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8. $6\times8=48$, and $\frac{48}{8}$ the Ans.—Proof. $\frac{48}{8}=48\div8=6$.

2. Reduce 15 to a fraction, whose denominator shall be 12.
Ans. $\frac{180}{12}$.

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$.

tients again in the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms.

Or, Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

Examples.

1. Reduce $\frac{144}{192}$ to its lowest terms.

$8\left\{\begin{matrix}(4)\ (3)\\ \frac{144}{192}=\frac{18}{24}=\frac{6}{8}=\frac{3}{4}\end{matrix}\right.$ the answer.

Or thus:

288)480(1
288
192)288(1
192
Com. mea. 96)192(2
192

Therefore 96 is the greatest common measure:

and $96\left\{\frac{144}{192}=\frac{3}{4}\right.$ the same as before.

2. Reduce $\frac{77}{616}$ to its lowest terms. Ans. $\frac{1}{8}$.
3. Reduce $\frac{44}{176}$ to its lowest terms. Ans. $\frac{1}{4}$.
4. Reduce $\frac{1111}{3333}$ to its lowest terms. Ans. $\frac{1}{3}$.

divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. When numbers with the sign of addition or subtraction between them, are to be divided by any numbers, each of the numbers must be divided: Thus, $\frac{6+9+12}{3}=2+3+4=9$.

2. But if the numbers have the sign of multiplication between them; then only one of them must be divided: Thus, $\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

36
×8+5
——
Ans. 293
——
8

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum, 293, is the numerator of the fraction sought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$

Or, $\frac{36\times8+5}{8}=\frac{293}{8}$ Ans. as before.

2. Reduce $127\frac{4}{7}$ to its equivalent improper fraction.
Ans. $\frac{893}{7}$.

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.
Ans. $\frac{12410}{19}$.

CASE III.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8. $6\times8=48$, and $\frac{48}{8}$ the Ans.—Proof. $\frac{48}{8}=48\div8=6$.

2. Reduce 15 to a fraction, whose denominator shall be 12.
Ans. $\frac{180}{12}$.

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$.

CASE IV.*

To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

Divide the numerator by the denominator; the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

EXAMPLES.

1. Reduce $\frac{293}{8}$ to its equivalent whole or mixed number.

8)293(36$\frac{5}{8}$ Ans.
24
53
48
5

Or, $\frac{293}{8}=293\div 8=36\frac{5}{8}$ as before.

2. Reduce $\frac{2166}{17}$ to its equivalent whole or mixed number.

Ans. $127\frac{7}{17}$.

3. Reduce $\frac{45}{5}$ to its equivalent whole number.

Ans. 9.

CASE V.

To reduce a compound fraction to an equivalent simple one.

RULE.

Multiply all the numerators continually together for a new numerator, and all the denominators for a new denominator, and they will form the simple fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction, by case 2d or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged, and the other members continually multi-

* This case is evidently the reverse of Case 2d.

plied, (as by the rule) will produce the fractions required in lower terms.

Examples.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

$$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5} \text{ the Ans.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction.

$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24}$ Ans. Or, by expunging the equal numerators and denominators, it will be $\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24}$ as before.

2. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple fraction.

Ans. $\frac{75}{64}=1\frac{11}{64}$.

CASE VI.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

Rule I.

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators into each other continually, for a common denominator.

Examples.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$1\times5\times8=40$ the new numerator for $\frac{1}{4}$.
$2\times4\times8=64$ ditto. $\frac{2}{5}$.
$5\times4\times5=100$ ditto. $\frac{5}{8}$.
$4\times5\times8=160$ the common denominator.

Therefore the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$, and $\frac{100}{160}$, the answer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{6}$, and $\frac{7}{8}$, to fractions, having a common denominator.

Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, of $\frac{4}{5}$, $7\frac{3}{4}$, and $\frac{5}{13}$ to a common denominator. Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{1404}{1872}$, $\frac{432}{1872}$.

Rule II.

To reduce any given fractions to others, which shall have the least common denominator.

1. By Problem 2, (page 56) find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

Examples.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$ to fractions having the least common denominator possible.

4)3 4 8

3 1 2

$4\times3\times2=24$=least common denomminator.

$\overline{24\div3}\times1=8$ the first numerator; $\overline{24\div4}\times3=18$ the second numerator; $\overline{24\div8}\times7=21$ the third numerator.

Whence the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions having the least common denominator. Ans. $\frac{30}{60}$, $\frac{24}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

CASE VII.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

Rule.

Reduce the given fraction to a compound one by comparing it with all the denominations between it and that denomination you would reduce it to; lastly, reduce this compound fraction to a single one, by Case V. and you will have a fraction of the required denomination, equal in value to the given fraction.

EXAMPLES.

1. Reduce $\frac{4}{7}$ of a cent to the fraction of a dollar.
By comparing it, it becomes $\frac{4}{7}$ of $\frac{1}{10}$ of $\frac{1}{10}$, which reduced by Case V. will be $\frac{4\times1\times1}{7\times10\times10}=\frac{4}{700}=\frac{1}{175}$ Dol. Ans.

2. Reduce $\frac{3}{8}$ of a penny to the fraction of a pound.
Ans. $\pounds\frac{1}{1\,0\,6}$.

3. Reduce $\frac{4}{7}$ of an ounce to the fraction of a pound Avoirdupois. Ans. $\frac{1}{28}$lb.

4. Reduce $\frac{7}{8}$ of a pennyweight to the fraction of a lb. Troy. Ans. $\frac{7}{1\,9\,2\,0}$lb.

5. Reduce $\frac{8}{9}$ of a lb. Avoirdupois to the fraction of a Cwt. Ans. $\frac{1}{1\,2\,8}$ Cwt.

CASE VIII.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

RULE.

Multiply the given numerator by the parts of the denominations between it and that denomination you would reduce it to, for a new numerator, which place over the given denominator: Or, only invert the parts contained in the integer, and make of them a compound fraction as before; then reduce it to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{175}$ of a dollar to the fraction of a cent.
By comparing the fraction it will be $\frac{1}{175}$ of $\frac{10}{1}$ of $\frac{10}{1}$;
then $\frac{1\times10\times10}{175\times1\times1}=\frac{100}{175}=\frac{4}{7}$ c. Ans.

2. Reduce $\frac{1}{4\,8\,0}$ of a pound to the fraction of a penny.
Ans. $\frac{3}{5}$d.

3. Reduce $\frac{1}{28}$ of a lb. Avoirdupois to the fraction of an ounce. Ans. $\frac{4}{7}$oz.

4. Reduce $\frac{7}{1\,9\,2\,0}$ of a lb. Troy to the fraction of a pwt.
Ans. $\frac{7}{8}$pwt.

5. Reduce $\frac{1}{1\,2\,0}$ of a cwt. to the fraction of a lb. Avoirdupois. Ans. $\frac{8}{9}$lb.

CASE IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, measure, &c.

RULE.

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on as far as necessary; and the quotients placed after one another, in their order, will be the answer required. Or, Reduce the numerator, as if it were a whole number, to the lowest denomination, and divide the result by the denominator; the quotient will be the number of the lowest denomination, (which must be brought into higher denominations as far as it will go) and the remainder will be a numerator to be placed over the given denominator for a fraction of the lowest denomination.

NOTE. From this rule, in connection with what has been said of Reduction of Federal Money, it appears, that annexing to the given numerator as many cyphers, as will fill all the places to the lowest denomination, and dividing the number so formed by the denominator, the quotient will be the answer in the several denominations, and the remainder a numerator to be placed over the given denominator, forming a fraction of the lowest denomination.

EXAMPLES.

1. What is the value of $\frac{5}{8}$ of a dollar?

By the general rule.

```
  5
 10
 --
8)50( 2
 --
d. 6  10
      --
   8)20( 4
   --
c. 2  10
      --
   8)40
   --
   m. 5
```

By the note.

```
D. d. c. m.
8)5 0 0 0
-------
    6 2 5
```

Ans. 6d. 2c. 5m.
or 62c. 5m.

Or thus.

5D.=5000m. and $\frac{5000}{64}$m.=625m.=62c. 5m. answer, as before.

2. What is the value of $\frac{17}{64}$ of a dollar?

D.	d.	c.	m.	
64)17	0	0	0	(2d. 6c. $5\frac{5}{8}$m.
128				
420				or 26c. $5\frac{5}{8}$m.
384				Or, 17D.=1700m. And
360				$\frac{17000}{64}$m=$265\frac{5}{8}$m.=
320				26c. $5\frac{5}{8}$m. Ans. as before.
40				
$\frac{40}{64}=\frac{5}{8}$				

3. What is the value of $\frac{5}{7}$ of a pound?

Ans. 14s. 3d. $1\frac{1}{7}$qr.

4. What is the value of $\frac{1}{3}$ a lb. Avoirdupois?

Ans. 12oz. $12\frac{1}{2}$dr.

5. What is the value of $\frac{5}{6}$ of a mile?

Ans. 6fur. 26pol. 11ft.

6. What is the value of $\frac{9}{13}$ of a day?

Ans. 16h. 36m. $55\frac{5}{13}$s.

7. What is the value of $\frac{3}{7}$ of an acre?

Ans. 3roods $17\frac{1}{7}$p.

CASE X.*

To reduce any given quantity to the fraction of any greater denomination of the same kind.

Rule.

Reduce the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term for a denominator; which will be the fraction required.

* This case is the reverse of the former, therefore proves it.

Note. If there be a fraction given with the said quantity, it must be farther reduced to the denominative parts thereof, adding thereto the numerator.

NOTE. It appears from this rule, and what has been said before, that in Federal Money, where the given quantity contains no fraction of its lowest denomination, the annexing of as many cyphers to 1 of the required denomination, as will extend to the lowest denomination in the given quantity, will form a denominator, which placed under the given quantity used as one number for a numerator, will make the answer, which may be reduced to its lowest terms. Or, if there be a fraction of the lowest denomination, multiply the given whole numbers by its denominator, adding its numerator, for a numerator; and let the denominator itself, at the left of as many cyphers as were mentioned above, be a denominator; the fraction so formed will be the answer; which may be reduced to its lowest terms.

EXAMPLES.

1. Reduce 6d. 2c. 5m. to the fraction of a dollar.

By the general rule.

6d.	10d. int. part.
×10+2	10
62	100
×10+5	10
625	1000

And $\frac{625}{1000}=\frac{5}{8}$D. Ans.

By the note.

D. d. c. m.

6 2 5

1 0 0 0 $=\frac{5}{8}$D.

Ans. as before.

2. Reduce 26c. $5\frac{1}{2}$m. to the fraction of a dollar.

By the general rule.

26c.	100c. int. pt.
×10+5	10
265	1000
×8+5	8
2125	8000

And $\frac{2125}{8000}=\frac{17}{64}$D. Ans.

By the note.

D. d. c. m.

$265\times8+5=2\ 1\ 2\ 5=\frac{17}{64}$D.

And $1\text{D.}\times8=8\ 0\ 0\ 0$

Ans. as before.

3. Reduce 14s. 3d. 1qr. $\frac{4}{7}$ to the fraction of a pound.

Ans. $\frac{4\,3\,9\,9}{7\,2\,0\,0}=\frac{5}{7}$£.

4. Reduce 12oz. 12$\frac{1}{2}$dr. to the fraction of a lb. Avoirdupois. Ans. $\frac{1}{3}$lb.

5. Reduce 6fur. 26po. 11ft. to the fraction of a mile.

Ans. $\frac{4}{5}$m.

6. Reduce 16h. 36m. 55$\frac{5}{11}$s. to the fraction of a day.

Ans. $\frac{9}{11}$ day.

7. Reduce 3r. 17$\frac{1}{2}$po. to the fraction of an acre.

Ans. $\frac{4}{7}$ acre.

ADDITION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then, the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. Add 7$\frac{1}{2}$, $\frac{4}{7}$ of $\frac{3}{5}$, and 7 together.

First. $7\frac{1}{2}=\frac{15}{2}$, $\frac{4}{7}$ of $\frac{3}{5}=\frac{12}{35}$, and $7=\frac{7}{1}$.

Then the fractions are $\frac{15}{2}$, $\frac{12}{35}$, and $\frac{7}{1}$; therefore,

$39\times56\times1=2184$

$15\times5\times1=75$

$7\times5\times56=1960$

4219

$5\times56\times1=280$

$\frac{4219}{280}=15\frac{19}{280}$.

Or thus, $\frac{2184+75+1960}{280}=15\frac{19}{280}$.

2. What is the sum of $\frac{7}{10}$ of $4\frac{4}{9}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $8\frac{1}{4}$?

Ans. $12\frac{5}{6}\frac{2}{0}$.

3. Add together $\frac{1}{3}$D, $\frac{4}{5}$c. $\frac{3}{10}$c. and $\frac{7}{8}$m

Ans. 20c. 9m.

4. Add $\frac{1}{8}$l. $\frac{3}{7}$s. and $\frac{1}{8}$d. together. Ans. 2s. $8\frac{54}{105}$d.

5. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together.

Ans. 2 days, 2 hours, 30 minutes, 45 seconds.

SUBTRACTION OF VULGAR FRACTIONS.

Rule.*

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

Examples.

1. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{6}$.

$\frac{2}{7}$ of $\frac{5}{6}=\frac{10}{42}=\frac{5}{21}$. Then the fractions are $\frac{3}{4}$ and $\frac{5}{21}$.

$3\times 28= 84$
$5\times 4= 20$
$4\times 28=112$ com. den.

$\frac{3}{4}=\frac{84}{112}$, and $\frac{5}{28}=\frac{20}{112}$, therefore, $\frac{84}{112}-\frac{20}{112}=\frac{64}{112}=\frac{4}{7}$ remainder.

2. From $\frac{49}{50}$ take $\frac{5}{9}$. Ans. $\frac{191}{450}$.

* In subtracting mixed numbers, when the fractions have a common denominator, and the numerator in the subtrahend is less than that in the minuend, the difference of the whole numbers will be a whole number, and the difference of the numerators a numerator to be placed over the given denominator: this whole number and the fraction thus formed will be the remainder: but, when the numerator in the subtrahend is greater than that in the minuend, subtract the numerator in the subtrahend from the common denominator, adding the numerator in the minuend, and carrying one to the integer of the subtrahend.

Hence, A fraction is subtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

Examples.

From	$12\frac{2}{5}$	$12\frac{2}{5}$	12
Take	$7\frac{3}{5}$	$7\frac{3}{5}$	$\frac{3}{5}$
Rem.	$5\frac{4}{5}$	$4\frac{4}{5}$	$11\frac{2}{5}$

3. Take $3\frac{1}{4}$c. from $\frac{1}{3}$ of $2\frac{1}{2}$D. Ans. $43\frac{1}{4}$c.

4. From $\frac{1}{4}$£. take $\frac{9}{10}$s. Ans. 4s. $1\frac{1}{2}$d.

5. From 5 weeks take $19\frac{1}{2}$ days.

Ans. 15da. 4ho. 48min.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.

NOTE. Where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

EXAMPLES.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$4\frac{1}{3}=\frac{13}{3}$, $\frac{1}{4}$ of $\frac{7}{8}=\frac{1\times7}{4\times8}=\frac{7}{32}$ and $6=\frac{6}{1}$.

Then $\frac{13}{3}\times\frac{1}{5}\times\frac{7}{32}\times\frac{6}{1}=\frac{13\times1\times7\times6}{3\times5\times32\times1}=\frac{546}{480}=1\frac{11}{160}$ the answer.

2. Multiply $\frac{4}{17}$ by $\frac{5}{27}$. Ans. $\frac{20}{459}$.

3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans. $\frac{7}{8}$.

4. Multiply $\frac{1}{3}$ of 5 by $\frac{2}{7}$ of $\frac{3}{7}$. Ans. $\frac{5}{49}$.

[☞See a method for the multiplication of mixed quantities, on page 6, which should have been inserted here.]

DIVISION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as before; then, invert the divisor, and proceed exactly as in Multiplication: The products will be the quotient required.

* To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

EXAMPLES.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{3}{4}$.

$\frac{1}{3}$ of $17 = \frac{1}{3}$ of $17 = \frac{1 \times 17}{3 \times 1} = 17/3$, and $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12} = \frac{1}{2}$; therefore, $17/3 \div \frac{1}{2} = \frac{17 \times 2}{3 \times 1} = \frac{34}{3} = 11\frac{1}{3}$ the quotient required.

2. Divide $\frac{4}{7}$ by $\frac{3}{5}$. Ans. $1\frac{8}{21}$.
3. Divide $5\frac{1}{2}$ by $7\frac{3}{4}$. Ans. $\frac{44}{62}$.
4. Divide $\frac{2}{3}$ by 9. Ans. $\frac{2}{27}$.
5. Divide $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{1}{2}$ of $\frac{3}{4}$. Ans. $\frac{4}{9}$.

DECIMAL FRACTIONS.

DECIMAL Fractions are of such a nature, that they vary in the same proportion, and are managed by the same method of operation, as whole numbers are.

On this account, every proper fraction is supposed to be reducible to another, whose denominator shall be 10, 100, 1000, &c. viz. Unity with a number of cyphers annexed; and fractions with such denominators are called *Decimal Fractions:* Such are $\frac{5}{10}$, $\frac{55}{100}$, $\frac{675}{1000}$, &c.

As the denominator of a decimal fraction is always 10, 100, 1000, &c. the denominators need not be expressed: For the numerator only may be made to express the true value: For this purpose it is only required to write the numerator with a point before it, at the left hand, to distinguish it from a whole number; when it consists of so many figures, as the denominator hath cyphers annexed to unity, or 1: So $\frac{5}{10}$ is written ·5; $\frac{33}{100}$, ·33; $\frac{735}{1000}$, ·735, &c.

NOTE. The point prefixed is called a Separatrix.

But if the numerator has not so many places as the denominator has cyphers, put so many cyphers before it, viz. at the left hand, as will make up the defect: so write $\frac{5}{100}$ thus, ·05; and $\frac{6}{1000}$ thus, ·006, &c. And thus do these fractions receive the form of whole numbers.

The 1st, 2d, 3d, 4th, &c. places of decimals, counting from the left hand toward the right, are called primes, seconds, thirds, fourths, &c.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand to 0, as in the following

TABLE.*

C Millians	X Millions	Millions	C Thousands	X Thousands	Thousands	Hundreds	Tens	Units	Tenth Parts *	Hundredth Parts	Thousandth Parts	X Thousandth Parts	C Thousandth Parts	Millionth Parts	X Millionth Parts	C Millionth Parts
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9

* It will be very apparent to the learner from the nature of decimals, and what has been said of Federal Money, that this money is purely decimal; and the dollar being the money unit, the lower denominations are plainly so many decimal parts of a dollar; thus 9 dollars and 8 dimes are expressed $9{\cdot}8 = 9\frac{8}{10}$ doll.—12 dollars, 4 dimes and 7 cents thus, $12{\cdot}47 = 12\frac{47}{100}$ doll.—20 dollars, 3 dimes, 4 cents, and 5 mills, thus, $20{\cdot}345 = 20\frac{345}{1000}$ dollars—100 dollars and 9 mills, thus, $100{\cdot}009 = 100\frac{9}{1000}$ doll.—50 dollars and 5 cents, thus, $50{\cdot}05 = 50\frac{5}{100}$ doll.; wherefore, it is, in all respects, added, subtracted, multiplied, and divided, the same as decimals; and, of all coins, it is the most simple.

It may also be observed, that the sum exhibits the particular number of each different piece of money contained in it, viz. 455997 mills $= 45599\frac{7}{10}$ cents $= 4559\frac{97}{100}$ dimes $= 455\frac{997}{1000}$ dolls. =

E. D. d. c. m.

$45\frac{5997}{10000}$ eagles = 45 5 9 9 7.

Also the names of the coins, less than a dollar, are significant of their values. For the *mill*, which stands in the third place at the right hand of the separatrix, or place of thousandths, is contracted from *mille*, the Latin for *thousand*: *Cent*, which occupies the second place, or place of hundredths, is an abbreviation of *centum*, the Latin for *hundred*: And *dime*, which is in the first place, is derived from *dieme*, the French for *tenths*.

From this table it is evident, that, in decimals, as well as in whole numbers, each figure takes its value by its distance from unit's place: If it be in the first place after units (or the separating point) it signifies tenths; if in the second, hundreds, &c. decreasing in each place in a tenfold proportion.

Consequently, every single figure expressing a decimal has for its denominator an unit, or 1, with so many cyphers as its place is distant from units's place: Thus, 2 in the decimal part of the table $=\frac{2}{10}$; $3=\frac{3}{100}$; $4=\frac{4}{1000}$, &c. And if a decimal be expressed by several figures, the denominator is 1, with so many cyphers as the lowest figure is distant from its unit's place.* So ·357 signifies $\frac{357}{1000}$, and $\cdot0053=\frac{53}{10000}$, &c.

Cyphers placed at the right hand of a decimal fraction do not alter its value, since every significant figure continues to possess the same place: So ·5, ·50, and ·500, are all of the same value, and each equal to $\frac{1}{2}$.

But cyphers placed at the left hand of a decimal, do alter its value, every cypher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place further from the place of units. So ·5, ·05, ·005, all express different decimals, viz. ·5, $\frac{5}{10}$; ·05, $\frac{5}{100}$; ·005, $\frac{5}{1000}$.

Hence may be observed the contrary effects of cyphers being annexed to whole numbers and decimals.

It is likewise evident from the table, that since the places of decimals decrease in a tenfold proportion from units downwards, so they consequently increase in a tenfold proportion from the right hand toward the left, as the places of whole numbers do: For, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $\frac{10}{100}=\frac{1}{10}$, $\frac{10}{10}=1$, and $1\times10=10$, which proves that decimals are subject to the same law of Notation, and consequently of operation, as whole numbers are.

Decimal fractions of unequal denominators are reduced to one common denominator, when there are annexed

* Such being the nature of Federal Money, its operations can in no other way be so well understood, as in obtaining a good knowledge of decimals, and applying their several rules to the various cases of money matters.

to the right hand of those, which have fewer places, so many cyphers as make them equal in places with that which has the most. So these decimals, ·5, ·06, ·455, may be reduced to the decimals ·500, ·060, ·455, which have all 1000 for their denominator.

Of decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places: Thus, ·5 is greater than ·459, for if it be reduced to the same denominator with ·459, it will be ·500.

A mixed number, viz. a whole number with a decimal annexed, is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number, and the denominator that of the decimal part. So 45·309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction: Thus, $45\times1000+309=\frac{45309}{1000}$ as above.

ADDITION OF DECIMALS.

Rule.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their sum, as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Examples.

1. Find the sum of 19·073+2·3597+223+·0197581+3478·1+12·358.

```
  19·073
   2·3597
 223·
   ·0197581
3478·1
  12·358
-----------
3734·9104581   the sum.
-----------
```

2. Required the sum of 429+21·37+355·003+1·07+1·7?
Ans. 808·143.

3. Required the sum of 973+19+1·75+93·7164+·9501?
Ans. 1088·4165.

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value: Then subtract as in whole numbers, and point off the decimals as in Addition.

EXAMPLES.

1. Find the difference of 1793·13 and 817·05693.

From 1793·13
Take 817·05693

Remainder 976·07307

2. From 171·195 take 125·9176 Ans. 45·2774.
3. From 219·1384 take 195·91. Ans. 23·2284.
4. From 480 take 245·0075. Ans. 234·9925.

MULTIPLICATION OF DECIMALS.

RULE.

1. Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. Point off so many figures from the product as there are decimal places in both the factors; and if there be not so many places in the product, supply the defect by prefixing cyphers.

EXAMPLES.

1. Multiply ·02345
by ·00163

7035
14070
2345

·0000382235 the product.

2. Multiply 25·138 by 12·17. Ans. 307·14646.
3. Multiply ·3759 by ·945. Ans. ·3552255.
4. Multiply ·84179 by ·0385. Ans. ·032408915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has cyphers.

So ·345 multiplied by $\left\{\begin{matrix}10\\100\\1000\end{matrix}\right\}$ makes $\left\{\begin{matrix}3·45\\34·5\\345\end{matrix}\right\}$

For ·345×10 is 3·450, &c.

DIVISION OF DECIMALS.

RULE.

1. The places of decimal parts in the divisor and quotient, counted together, must always be equal to those in the dividend; therefore, divide as in whole numbers, and, from the right hand of the quotient point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend or to the remainder, and the quotient carried on to any degree of exactness.

EXAMPLES.

1.

219).117841075(·000538087, &c.
1095

834
657

1771
1752

1907
1752

1555
1533

22

2.

·3719)38·0000(102·178, &c.
3719

8100
7438

6620
3719

29010
26033

29770
29752

18

In example 1st. the divisor having no decimals, the quotient must have so many as there are in the dividend. In Example 2d, the dividend being an integer, must have at least so many cyphers annexed, as there are decimals in the divisor, and so far the quotient will be whole numbers; then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the rule.

3. 133)5737(43·1353+
4. 23·7)65321(2756·16+
5. *72)918·217(12753+
6. 25·17)315·6293(1253+
7. ·317)29·417(92+

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers) it is performed by removing the separatrix, in the dividend, so many places toward the left hand, as there are cyphers in the divisor.

So $\left\{\begin{matrix}10\\100\\1000\\10000\end{matrix}\right\}$ dividing 7654, the quotient is $\left\{\begin{matrix}765·4\\76·54\\7·654\\·7654\end{matrix}\right\}$

* The following questions are left unpointed in the quotient, to exercise the learner.

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal.

RULE.

Divide the numerator by the denominator, as in Division of Decimals, and the quotient will be the decimal required. Or, So many cyphers as you annex to the given numerator, so many places must be pointed off in the quotient, and if there be not so many places of figures in the quotient, the deficiency must be supplied by prefixing so many cyphers before the quotient figures.

EXAMPLES

1. Reduce $\frac{1}{8}$ to a decimal. 8)1·000

·125 Ans.

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{2}{3}$ to decimals.
Ans. ·375, ·625, ·666+.

3. Reduce $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$, and $\frac{7}{8}$ to decimals.
Ans. ·25, ·5, ·75, ·333+, ·8, ·833+, ·875.

4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{456}$, and $\frac{9}{36}$, to decimals.
Ans. ·263+, ·692+, ·025, ·25.

CASE II.

To reduce numbers of different denominations, as of Money, Weights, and Measure, to their equivalent decimal values.

RULE.

1. Write the given numbers perpendicularly under each other, for dividends, proceeding orderly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number, for a divisor, as will bring it to the next superior denomination, and draw a line perpendicularly between them.

3. Begin with the highest, and write the quotient of each division as decimal parts on the right hand of the dividend next below it, and so on, until they are all used, and the last quotient will be the decimal sought.

EXAMPLES.

1. Reduce 17s. 8¾d. to the decimal of a pound.

4	3·
12	8·75
20	17·720166, &c.

·886458, &c. the decimal required.

Here, in dividing 3 by 4, I suppose two cyphers to be annexed to the 3, which makes it 3·00, and ·75 is the quotient, which I write against 8 in the next line: this quotient, viz. 8·75, being pence and decimal parts of a penny, I divide them by 12, which brings them to shillings and decimal parts; I therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce 1, 2, 3, 4, and so on to 19 shillings, to decimals.

Shillings.	1	2	3	4	5	6	
Answers.	·05,	·1,	·15,	·2,	·25,	·3,	
Shillings.	7	8	9	10	11	12	
Answers.	·35,	·4,	·45,	·5,	·55.	·6,	
Shillings,	13	14	15	16	17	18	19
Ans.	·65,	·7,	·75,	·8,	·85,	·9,	·95.

Here, when the shillings are even, half the number, with a point prefixed, is their decimal expression; but if the number be odd, annex a cypher to the shillings, and then halving them, you will have their decimal expression.

3. * Reduce 1, 2, 3, and so on to 11 pence, to the decimals of a shilling.

Pence.	1	2	3	4	5	6
Answers.	·083+,	·166+,	·25,	·333+,	·416+,	·5,
Pence.	7	8	9	10	11	
Answers.	·583+,	·666+,	·75,	·833+,	·916+.	

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

* The answers to this question are the same as the decimal parts of a foot.

Pence.	1	2	3	4
Answers.	·00416+,	·0083+,	·0125	·01666+.
Pence.	5	6	7	8
Answers.	·0208+,	·025,	·02916+,	·0333+,
Pence,	9	10		11
Answers.	·0375,	·0416+,		·04583+.

5. Reduce 1, 2, and 3 farthings to the decimals of a penny. Ans. 1qr.=·25d. 2qrs.=·5d. and 3qrs.=·75d.

6. Reduce 1, 2, and 3 farthings to the decimals of a shilling. Ans. 1qr.=·02083+s. 2qrs.=·04166+s. 3qrs.=·0625s.

7. Reduce 1, 2, and 3 farthings to the decimals of a pound. Ans. 1qr.=£·0010416+ 2qrs.=£·002083+ 3qrs.=£·003125.

8. Reduce 13s. 5½d. to the decimal of a pound. Ans. ·6729+.

9. Reduce 7ewt. 3qrs. 17lb. 10oz. 12dr. to the decimal of a ton. Ans. ·39538+.

10. Reduce 10oz. 13pwt. 9gr. to the decimal of a pound Troy. Ans. ·8890625.

11. Reduce 3qrs. 3n. to the decimal of a yard. Ans. ·9375.

CASE III.

To find the decimal of any number of shillings, pence, and farthings, by inspection.

Rule.

1. Write half the greatest even number of shillings for the first decimal figure.

2. Let the farthings in the given pence and farthings possess the second and third places; observing to increase the second place, or place of hundredths, by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

Examples.

1. Find the decimal of 13s. 9¾d. by inspection.

·6 . . = ½ of 12s.
5 for the odd shilling.
39 = farthings in 9¾d.
Add 2 for the excess of 36.

·691 = decimal required.

2. Find by inspection the decimal expressions of 18s. 3¼d. and 17s. 8¼d. Ans. £·914 and £·885.

CASE IV.

To find the value of any given decimal in the terms of the integer.

RULE.—1. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferior denomination, and cut off a remainder as before.

3. Proceed in this manner through all the parts of the integer, and the several denominations, standing on the left hand, make the answer.

EXAMPLES.

1. Find the value of ·73968 of a pound.

20

14·79360

12

9·52320

4

2·09280 Ans. 14s. 9½d.

2. What is the value of ·679 of a shilling
Ans. 8·148d.

3. What is the value of ·617 of a cwt.?
Ans. 2qrs. 13lb. 1oz. 10·624dr.

4. What is the value of ·397 of a yard?
Ans. 1qr. 2·352n.

5. What is the value of ·8469 of a degree?
Ans. 58m. 6fur. 35po. 0ft. 11in.

6. What is the value of ·569 of a year?
Ans. 207d. 16h. 26m. 24 sec.

CASE V.

To find the value of any decimal of a pound by inspection.

Rule.

Double the first figure, or place of tenths, for shillings, and if the second figure be 5, or more than 5, reckon another shilling; then, after the 5 is deducted, call the figures in the second and third places so many farthings, abating 1 when they are above 12, and 2 when above 36, and the result will be the answer.

Note. When the decimal has but two figures, if any thing remain after the shillings are taken out, a cypher must be annexed to the right hand, or supposed to be so.

Examples.

1. Find the value of £·876 by inspection.

16s.=double of 8
1s. for the 5 in the second place, which is to be taken out of 7,
and 6¼d.=26 farthings, remain to be added;
deduct ¼d. for the excess of 12.

17s. 6¼d. Ans.

2. Find by inspection, the value £·49.

8s.=double of 4.
1s. for the 5 in the place of hundredths.
10.=40 farthings, a 0 being annexed to the remaining 4.
Ded. ½d. for the excess of 36.

9s. 9½d. the answer.

3. Find the value of £·097 by inspection.

Ans. 1s. 11¼d.

4. Value the following decimals, by inspection, and find their sum, viz. £·785+£·537+£·916+£·74+£·5+£·25+£·09+£·008.

Ans. £3 18s. 0d.

Decimal Tables of Coin, Weight, and Measure.

TABLE I. Coin. £1 the Integer.

Shil.	dec	Shil.	dec
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

Pence.	Decimals.
11	·045833
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Farth.	Decimals.
3	·003125
2	·0020833
1	·0010416

TABLE II. Coin & Long Meas. 1 Shilling & 1 Foot the Integer.

Pence & Inches.	Decimals
11	·916666
10	833333
9	·75
8	·666666
7	583333
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Farthings	Decimals.
3	·0625
2	·041666
1	·020833

TABLE III. Troy Weight. 1 Pound the Integer. Ounces the same as Tab. II.

Penny-weights.	Decimals
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Grains.	Decimals.
12	·002083
11	·00191
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 Oz. the Integer. Pennyweights the same as shillings in Tab. I.

Grains.	Decimals.
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125
5	·010416
4	·008333
3	·00625
2	·004165
1	·002083

TABLE IV. Avoirdupois Wt. 112lb. the Integer

Qrs.	Decimals.
3	·75
2	·5
1	·25

Pounds.	Decimals.
27	·241071
26	·232143
25	·223214
24	·214286
23	·205357
22	·196428
21	·1875
20	·178571
19	·169643
18	·160714
17	·151786
16	·142857
15	·133928
14	·125
13	·116071
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Ounces.	Decimals.
15	·008370
14	·007812
13	·007254
12	·006696

Carried Over.

Brought over.	
Ounces.	Decimals.
11	·006138
10	·00558
9	·005022
8	·004464
7	·003906
6	·003348
5	·00279
4	·002232
3	·001674
2	·001116
1	·000558
qrs.of oz.	Decimals
3	·000418
2	·000279
1	000139

TABLE V.
Avoirdupois Wt.
1lb. the Integer.

Ounces.	Decimals.
15	·9375
14	·875
13	·8125
12	·75
11	·6875
10	·625
9	·5625
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625
Drams.	Decimals.
15	·058594
14	·054687
13	·050781
12	·046875
11	·042969
10	·039062
9	·035156
8	·03125
7	·027344
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.
Cloth Measure.
1 Yard the Integer.

Quarters	Decimals.
3	·75
2	·5
1	·25
Nails.	Decimals.
3	·1875
2	·125
1	·0625

TABLE VII.
Long Measure.
1 Mile the Integer.

Yards.	Decimals.
1000	·568181
900	·511354
800	·454545
700	397727
600	·340909
500	·284091
400	·227272
300	·170455
200	·113636
100	·056818
90	·051136
80	·045454
70	·039773
60	·034091
50	·028409
40	·022727
30	·017045
20	·011364
10	·005682
9	·005114
8	·004545
7	·003977
6	·093409
5	·002841
4	·002273
3	·001705
2	·001136
1	·000568
Feet.	Decimals.
2	·0003787
1	·0001892
Inches.	Decimals.
6	·0000947
5	·000079
4	·0000632
3	·0000474
2	·0000316
1	·0000158

TABLE VIII.
Liquid Measure.
1 Gall. the Integer.
Quarts the same as qrs. in Tab. VI.

1 Pint	·125
3 Gill	·09375
2	·0625
1	·03125

TABLE IX.
Time.
1 Year the Integer.
Months the same as Pence in Tab. II.

Days.	Decimals.
365	1·000000
300	·821918
200	·547945
100	·273973
90	·246575
80	·219178
70	·191781
60	·164384
50	·136986
40	·109589

Carried over.

Brought over.	
Days.	Decimals.
30	·082192
20	·054795
10	·027397
9	·024658
8	·021918
7	·019178
6	·016438
5	·013699
4	·010959
3	·008419
2	·005479
1	·00274
1 Day the Integer.	
Hours.	Decimals.
23	·958333
22	·916666
21	·875
20	·833333
19	·791665
18	·75
17	·708333
16	·666666
15	·625
14	·583333
13	·541666
12	·5
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041667
Minutes.	Decimals.
30	·020833
20	·013889
10	·006944
9	·00625
8	·005555
7	·004861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001389
1	·000694

COMPOUND MULTIPLICATION

IS extremely useful in finding the value of goods, &c. And, as in Compound Addition, we carry from the lowest denomination to the next higher, so we begin and carry in Compound Multiplication; one general rule being to multiply the price by the quantity.

CASE I.

When the quantity does not exceed 12 *yards, pounds, &c.*—Set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and in multiplying by it, observe the same rules for carrying from one denomination to another, as in Compound Addition.

Introductory Examples.

	1.			2.			3.			
	£	s.	d.	£	s.	d.	D.	d.	c.	m.
Multiply	15	17	1	25	12	8	8	5	1	7
by			2			3				4

4.			5.		6.		
£.	s.	d.	D.	c.	£.	s.	d.
67	18	6½	4	75	13	12	11
		5		6			7

7.			8.				
£.	s.	d.	E.	D.	d.	c.	m.
31	16	8¼	2	7	8	9	1
		8					9

Practical Questions.

Note. The facility of reckoning in the Federal Money, compared with pounds, shillings, &c. may be seen from the examples in this and the following cases; where the same questions are given in both the currencies, as near as can be, avoiding small fractions. It may be observed, that the variety of cases here given, is applicable only to the *old* currency, while the same questions in the Federal are solved by plain decimals.

1. What will 9 yards of cloth at { 5s. 4d. / 88c. 9m. } per yard come to?

	£0	5	4	price of one yard,		·88c.	9m.
Multiplied by			9	yards			9
Ans.	£2	8	0	price of 9 yards.	D.	8 ·00	1

2. 3 yards at	15s. 4d. / 2D. 55c. 6m.	per. yard. =	£2 6s. / D.7 66c. 8m.		
3. 6 - -	9s. 10d. / D.1 63c. 9m.	- - =	£2 19s. / D.9 83c. 4m.		
4. 9 - -	13s. 7½d. / D.2 27c. 1m.	- - =	£6 2 7½ / D.20 ·439		

CASE II.

When the multiplier, that is, the quantity, is above 12: You must multiply by two such numbers, as, when multiplied together, will produce the given quantity.

EXAMPLES.

1. What will 144yds. of cloth cost at { 3s. 5½d. / 57c. 6¾m. } per yard?

	£	s.	d.		c.	m.
	0	3	5½	price of 1 yard.	·5764	
Multiply by			12		144	
Produces	2	1	6	price of 12 yards	23056	
Multiplied by			12		23056	
					5764	
Answer	£24	18	0	price of 144 yards.	Ans. D. 83·0016	

Questions.			Answers.
2. 24 yards, at	{ 6s. 3¾d. / D.1 5c. 2m. }	per yd. =	{ £7 11 6 / D.25·24 8 }
3. 27 ———	{ 9s. 10d. / D.1 63c. 9m. }	———	{ £13 5 6 / D.44·25 3 }
4. 44 ———	{ 12s. 4½d. / D.2 6c. 2½m. }	———	{ £27 4 6 / D.90·75 }

CASE III.

When the quantity is such a number, that no two numbers in the table will produce it exactly: Then multiply by two such numbers as come the nearest to it: and for the number wanting, multiply the given price of 1 yard by the said number of yards wanting, and add the products together for the answer; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

EXAMPLES.

1. What will 47 yards of cloth, at { 17s. 9d. / D.2 95c. 8m. } per yard, come to?

	£	s.	d.		D.
	0	17	9	price of 1 yard.	D.2·958
Multiplied by			5		47
Produces	4	8	9	price of 5 yards.	20706
Multiplied by			9		11832
Produces	39	18	9	price of 45	Ans. D.139·026
Add	1	15	6	price of 2 yards.	
Ans.	£41	14	3	price of 47 yards.	

NOTE. This may be performed by first finding the value of 48 yards, from which if you subtract the price of 1, the remainder will be the answer as above.

Questions.			Answers.
2. 75 yards at	5s. 7½d. / 93c. 7½m.	per. yd. =	£21 1 10½ / D.70·312½
3. 67½ ———	16s. 3¼d. / D.2 71c. 1⅘m.	———	£54 18 3¼ / D.183·046¼
4. 59 ———	9s. 7d. / D.1 59c. 7m.	———	£28 5 5 / D.94·223

CASE IV.

When the quantity is any number above the Multiplication Table: Multiply the price of 1 yard by 10, which will produce the price of 10 yards: *This* product, multiplied by 10, will give the price of 100 yards; then, you must multiply the price of 100 by the number of hundreds in your question; the price of 10 by the number of tens; and the price of unity, or 1, by the number of units: Lastly, add these several products together, and the sum will be the answer.

EXAMPLES.

1. What will 359 yards of cloth, at { 4s. 7½d. / 77c. 1m. } per yard, amount to?

£.	s.	d.	
0	4	7½	price of 1 yard.
		10	
2	6	3	price of 10 yards.
		10	
23	2	6	price of 100 yards.
		3	
69	7	6	price of 300 yards.
11	11	3	5 times the price of 10yds. = price of 50 yards.
2	1	7½	9 timed the price of 1yd. = price of 9 yards
£83	0	4½	Ans. price of 359 yards.

c. m.
·771
359

6939
3855
2313

Ans. D.276·789

Questions.			Answers.
2. 297 yds. at	17s. 3½d. / D.2 88c. 2m.	per yd.=	£256 15 7½ / D.855·954
3. 473 ——	9s. 11¼d / D.1 65c. 6¼m.	——	£235 0 5¼ / D.783·406¼
4. 512 ——	15s. 10d. / D.2 63c. 9m.	——	£405 6 8 / D.1351·168

CASE V.

To find the value of one hundred weight: As 112 is the gross hundred, so 112 farthings are =2s. 4d. and 112d.=9s. 4d.; therefore, if the price be farthings, or not more than 3d. multiply 2s. 4d. by the farthings in the price of 1lb.; or, if the price be pence, multiply 9s. 4d. by the pence in the price of 1lb. and, in either case, the product will be the answer.

EXAMPLES.

1. What will 1cwt. of chalk come to, at { 1½d. / 2c. 1m. } per pound?

112 farthings=£0 2 4 price of 1cwt. at 1qr. per lb.
1½d. = 6 farthings in the price.

Answer £0 14 0 price of 1cwt. at 1½d. per lb.
·021
112

42
21
21

Answer D.2·352

2. 1Cwt of Tin, at $\left\{\begin{matrix}2\frac{1}{4}\text{d} \\ 3\text{c. } 1\frac{1}{4}\text{m.}\end{matrix}\right\}$ per pound?

·03125 2s. 4d. price of 1cwt. at ¼d. per lb.
112 9 farthings in the price of 1lb.

6250 £1 1 0 price of 1cwt. at 2¼d. per pound,
3125 answer.
3125

D,3·50000 Ans,

3. 1Cwt. of lead, at $\left\{\begin{matrix}7\text{d.} \\ 9\text{c. } 8\text{m.}\end{matrix}\right\}$ per pound?

·098 9s. 4d. price of 1cwt. at 1d. per lb.
112 7 pence in the price of 1lb.

196 £3 5 4 price of 1cwt. at 7d. per lb.
98
98

D. 10·976 Ans.

	Questions.			Answers.
4.	1Cwt. at $\left\{\begin{matrix}0\frac{3}{4}\text{d.} \\ 1\text{c. } 0{\cdot}4\text{m.}\end{matrix}\right\}$	per pound =		$\left\{\begin{matrix}£0\ \ 7s. \\ D.\ 1{\cdot}155\end{matrix}\right\}$
5.	—— $\left\{\begin{matrix}2\frac{1}{2}\text{d. } \frac{1}{2}\text{qr.} \\ 3\text{c. } 6\frac{1}{2}\text{m.}\end{matrix}\right\}$	—— =		$\left\{\begin{matrix}£1\ \ 4\ \ 6 \\ D.4{\cdot}088\end{matrix}\right\}$
6.	—— $\left\{\begin{matrix}4\frac{1}{2}\text{d.} \\ 6\frac{1}{4}\text{c.}\end{matrix}\right\}$	—— =		$\left\{\begin{matrix}£2\ \ 2s \\ D.7\end{matrix}\right\}$

CASE VI.

To find the value of two, or more hundreds, by having the price of one pound: First, find the price of 1cwt. by the last case, and then proceed to find the value of the whole, by case 1 or 2, as the question may require.

Examples.

1. What is the value of 5¼cwt. of sugar, at { 6d. / 8c. 3⅓m. } per pound?

£	s.	d.	
0	9	4	price of 1cwt. at 1d. per lb.
		6	
2	16	0	price of ditto at 6d. per lb.
		5	
14	0	0	price of 5cwt.
1qr. = 0	14	0	price of ¼ cwt.
Ans. £14	14	0	price of 5¼cwt.

cwt.	qr.
5	1
4	
21	
28	
168	
42	
588 lb.	

D.
·083⅓
588
664
664
415
196
D.49·000 Ans.

Questions.			Answers.	
2. 4Cwt. of sugar, at	2½d. / 3⅓c.	per lb. =	£4 13 4 / D.15 68c.	
3. 8½ ———	5d. / 7c.	——— =	£19 16 8 / D.66 64c.	
4. 7 ———	4¾d. / 6c. 6m.	——— =	£15 10 4 / D.51·744	

CASE VII.

To find the value of a hundred weight, when the price of 1lb. *is any number of pounds and shillings; or shillings, pence, and farthings:* Multiply the price of 1lb. by 7, its product by 8, and *this* product by 2; which last product will be the answer required.

Examples.

What will 1cwt. of tobacco cost, at { 5s. 7½d. / 93c. 7½m. } per lb.?

£	s.	d.		D.
0	5	7½	price of 1lb.	·9375
		7		112
1	19	4½	price of 7lb.	18750
		8		9375
15	15	0	price of 56lb. or ½ cwt.	9375
		2		Ans. D.105·
£31	10	0	Ans. price of 112lb. or 1cwt.	

Questions.				Answers.
2. 1cwt. at	3s. 10½d. / 64c. 6m.	per lb. =		£21 14s. / D.72 35c. 2m.
3. 1 ——	9s. 6d. / D.1 58½c.	— =		£53 4s. / D.177 33½c.
4. 1 ——	16s. 11½d. / D.2 82c. 6m.	— =		£94 19 4 / D.316·512

Practical Questions in Weights and Measures.

1. What is the weight of 4 hogsheads of sugar, each weighing 7cwt. 3qrs. 19lb.? Ans. 31cwt. 2qrs. 20lb.

2. What is the weight of 6 chests of tea, each weighing 3cwt. 2qrs. 9lb.? Ans. 21cwt. 1qr. 26lb.

3. If I am possessed of 1½ dozen of silver spoons, each weighing 3oz. 5pwt.—2 dozen of tea spoons, each weighing 15pwt. 14gr.—3 silver cans, 9oz. 7pwt.—2 silver tankards, each 21oz. 15pwt.—and 6 silver porringers, each 11oz. 18pwt. what is the weight of the whole?

Ans. 18lb. 4oz. 3pwt.

4. In 35 pieces of cloth, each measuring 27¾ yards, how many yards? Ans. 971¼ yards.

5. How much brandy in 9 casks, each containing 45 gallons, 3qt. 1pt.? Ans. 412gal. 3qt. 1pt.

6. If I have 9 fields, each of which contains 12 acres, 2 roods, and 25 poles; how many acres are there in the whole?

Ans. 113ac. 3r. 25p.

COMPOUND DIVISION

IS the dividing of numbers of different denominations: in doing which, always begin at the highest, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and so on, till you have divided the whole, taking care to set down your quotient figures under their respective denominations.

Introductory Examples.

1.

	£	s.	d.	
Divide	549	17	9	by 5
Quotient	£109	19	6½	

2.

	D.	d.	c.	m.
3)14	1	9	6	

3.

	£	s.	d.
4)731	5	10⅓	

4.

	£	s.	d.
6)37	11	4¾	

5.

	D.	c.	m.
7)25	49	4	

6.

	T.	cwt.	qrs.	lb.	oz.	dr.
3)29	13	2	25	1	13	

7.

	lb.	oz.	pwt.	gr.
10)849	11	12	14	

Practical Questions.

The same note, which was given under Practical Questions in Compound Multiplication, is applicable in this place.

CASE I.

Having the price of any number of yards, &c. within the pence table, to find the price of unity, or 1 *yard:* If the quantity do not exceed 12, proceed by setting down the price, and dividing it by the quantity; which quotient will be the price of one yard, required: but if the quantity exceed 12, then divide by two such numbers, as, when multiplied together, will produce the quantity, and the last quotient will *be the* value of 1 yard, required.

NOTE. This Case proves the first and second Cases in Compound Multiplication.

EXAMPLES.

1. If 9 yards of cloth cost { £4 3s. 7½d / D.13 93¾c. } what is it per yard?

£	s.	d.
9)4	3	7½
0	9	3½ Ans.

D.
9)13·9375
D.1·5486+ Ans.

2. If 7 ells cost { £5 17s. 5d. / D.19 57c. } what cost 1 ell?

Ans. { 16s. 9¼d. / D.2·796

3. If 11 sheep cost { £6 5s. 9d. / D.20 95c. 8m. } what did each cost?

Ans. { 11s. 5¼d. / D.1·905+

4. If 12 gallons of rum cost { £8 11s. 9½d. / D.28 63c. 2m. } what is it per gallon?

Ans. { 14s. 3¾d. / D. 2·386

5. If 84 cows cost { £253 13s. / D.845 50c. } what is the price of each?

Ans. { £3 os. 4¾d. / D.10 6½c.

6. If 132 bushels of corn cost { £20 12s. 6d. / D.68 75c. } what is that per bushel?

Ans. { 3s. 1½d. / 52c. 1m.

CASE II.

Having the price of a hundred weight, to find the price of 1lb: Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1lb. required.

EXAMPLES.

1. If 1cwt. of Flax cost { £2 7s. 8d. / D.7 94c. 4m. } what is that per pound?

£	s.	d.
8)2	7	8
7)0	5	11½
2)0	0	10d. 0⅞qr.

0 0 5$\frac{3}{16}$d. price of 1lb.

D.
112)7·944(·071—
784
104

Ans. 7c. 1m. nearly.

2. At { £3 10s. / D.11 67c. } per cwt. what cost 1lb.?

Ans. { 7½d. / 10c. 4m. }

3. At { £6 6s. / D.21 } per cwt. what cost 1lb.?

Ans. { 1s. 1½d. / 18c. 7½m. }

4. At { £42 11s. 8d. / D.141 94c. 4m. } per cwt. what cost 1lb.?

Ans. { 7s. 7¼d. / D.1·267 }

CASE III.

Having the price of several hundred weight, to find the price per pound: Divide the whole price by the number of hundreds, which will give the price per cwt. and then proceed as in the last Case.

EXAMPLES.

1. If 5cwt. of sugar cost { £13 8s. 4d. / D.44 72c. 2m. } what is that per lb.

£	s.	d.	
5)13	8	4	
8) 2	13	8	price of 1cwt.
7) 0	6	8½	price of 14lb. or $\frac{1}{8}$ cwt.
2) 0	0	11½	price of 2lb. or $\frac{1}{56}$ cwt.
0	0	5¾	price of 1lb.

112lb. in a cwt.
5 cwt.

560)44·722(·079$\frac{7}{8}$—, or 8 cents nearly, Ans.
392

552
504

482
——=$\frac{7}{8}$ nearly.
560

2. If 8cwt. of cocoa cost { £15 17s. 4d. / D.52 88c. 9m. } what is that per lb.? Ans. { 4$\frac{1}{4}$d. / 5c. 9m.+ }

3. If 3$\frac{1}{4}$cwt. of sugar cost { £9 17s. 2d. / D.32 86c. } what is it per lb? Ans. { 6$\frac{1}{2}$d. / 9c.+ }

4. If 1$\frac{3}{4}$cwt. of cotton wool cost { £6 10s. 8d. / D.21 77c. 8m. } what is that per lb? Ans. { 8d. / 11c. 1m.+ }

NOTE. This Case proves the 6th in Compound Multiplication.

CASE IV.

Having the price of any number of yards, &c. to find the price of 1 *yard:* Divide the price by the quantity, beginning at the highest denomination, and, if any thing remain, reduce it into the next, and every inferiour denomination, and, at each reduction, divide as before, remembering each time to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

NOTE. If there be $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed; or, in Federal Money, work by decimals.

Examples.

1. If 95½lb. of figs cost { £16. 13s. 6¾d. / D.55 59c. 3¾m. } what are they per lb.?

Quantity=95½lb. Price=£16 13 6¾
Mult. by 4 4

Produces 382 for a divisor. Product £66 14 3 for a (dividend.

382)66 14 3(0 3 5¾ ¾¹⁸²₃₈₂per lb.
20

382)1334(3
1146

188
12

382)2259(5
1910

349
4

382)1396(3
1146

250

D.c.m.dec.c.m.
95½=95·5)55·59375(·58 2+Ans.
4775

7843
7640

20337
1910

1275

2. If 147 bushels of rye cost { £47. 12s. 6d. / D.158 76c. } what is it per bushel? Ans. { 6s. 5¾d. / D.1 8c.

3. If 33¼ yards of baize cost { £25 13s. 9½d. / D.85 63c. 2m. } what is it per yard? Ans. { 15s. 5¼d. ⁹⁵⁄₁₃₈ / D.2 57c. 5m.

Note. This proves the 3d and 4th Cases in Compound *Multiplication*.

PRACTICAL QUESTIONS IN MONEY.

1. Divide { 273l. 9s. 4d. / D.911. 55c. 5m. } among 5 men and 4 women, and give the men twice as much as the women.

Men. Wom.
5 and 4
Mult. by 2

10 shares.
Add 4 women's shares.

14 the number of equal shares in the whole=Divisor.

£.	s.	d.		£.	s.	d.	
Divide by 14)273	9	4(		19	10	8	=1 woman's share.
14						4	women.
133				78	2	8	=women's share.
126							
7				19	10	8	
20						2	
14)149(10				39	1	4	=1 man's share.
14						5	men
9				195	6	8	=men's share.
12				78	2	8	=women's share.
14)112(8				273	9	4	Proof
112							

D.
14)911·555(65·111+ =1 woman's share.
84 4 women.
71 260·444=women's share.
70
15 65·111+
14 2
15 130·222+=1 man's share.
14 5
15 651·111+=men's share.
14 260·444+=women's share.
1 911·555+ Proof.

2. Divide $\left\{\begin{array}{l}120l.\ 1s.\ 4d.\\ D.402\ 88c.\ 9m.\end{array}\right\}$ among 7 men and 7 women, and give the women 3 times as much as the men.

Ans. $\left\{\begin{array}{l}\left\{\begin{array}{l}4l.\ 6s.\ 4d.\\ D.14\ 38c.\ 9m.\end{array}\right\} = \text{a man's share.}\\ \left\{\begin{array}{l}12l.\ 19s.\\ D43.\ 16c.\ 7m.\end{array}\right\} = \text{a woman's share.}\end{array}\right.$

3. Divide $\left\{\begin{array}{l}39l.\ 12s.\ 5d.\\ D.132.\ 7c.\end{array}\right\}$ among 4 men, 6 women, and 9 boys: Give each man double to a woman, and each woman double to a boy.

Ans. $\left\{\begin{array}{l}\left\{\begin{array}{l}1l.\ 1s.\ 5d.\\ D.3\ 57c.—\end{array}\right\} = \text{a boy's share.}\\ \left\{\begin{array}{l}2l.\ 2s.\ 10d.\\ D.7\ 14c.—\end{array}\right\} = \text{a woman's share.}\\ \left\{\begin{array}{l}4l.\ 5s.\ 8d.\\ D.14\ 28c.\end{array}\right\} = \text{a man's share.}\end{array}\right.$

DUODECIMALS.

Or, CROSS MULTIPLICATION,

IS a Rule, made use of by workmen and artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches, and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are maked thus: inches or primes, (′), seconds (″), thirds (‴), fourths (⁗), &c.

This method of multiplying is not confined to *twelves*, but may be greatly extended: For any number, whether its inferior denominations decrease from the integer in the same ratio, or not, may be multiplied cross-wise; and for the better understanding of it, the learner must observe, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand; but if he multiplies by any number of an inferior denomination, the value of an unit in the product will be so much inferior to the value of an unit in

the multiplicand as an unit of the multiplier is less than an integer.

Thus, pounds, multiplied by pounds, are pounds; pounds, multiplied by shillings, are shillings, &c.; shillings, multiplied by shillings, are twentieths of a shilling; shillings, multiplied by pence, are twentieths of a penny; pence, multiplied by pence, are 240ths of a penny, &c.

RULE.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term, observing, in duodecimals, to carry an unit for every 12, from each lower denomination to its next superior, and for other numbers accordingly.

3. In the same manner, multiply all the multiplicand by the primes, or second denomination in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand.

4. Do the same with the seconds in the multiplier, setting the result of each term two places to the right hand of those in the multiplicand.

5. Proceed in like manner with all the rest of the denominations, and their sum will be the answer required.

EXAMPLES.

1. Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

F.	′		Or thus.
2	6		$2\frac{1}{2}$
2	6		$2\frac{1}{2}$
5	0		5
1	3	0	$1\frac{1}{4}$
Ans. 6	3		Ans. $6\frac{1}{4}$ square feet=6 feet 36

inches. So that the 3 is not 3 inches, but 36in. or $\frac{1}{4}$ of a square foot.

Or thus, 2·5
2·5
125
50
Ans. 6·25 square feet.

2. Multiply 9f. 8′ 6″, by 7f. 9′ 3″.

F.	′	″			
9	8	6			
7	9	3			
67	11	6			=Prod. by the feet in the multiplier.
7	3	4	6‴		=do. by the primes.
	2	5	1	6⁗	=do. by the seconds.
75	5	3	7	6	Answer.

3. How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inchss wide? Ans. 24ft. 10′ 11″

4. How many cubick feet in a stick of timber, 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick? Ans. 35ft. 6′ 8″ 6‴

5. Multiply 3l. 6s. 8d. by 2l. 5s. 7d.

£.	s.	d.
3	6	8
2	5	7

£3×£2=£6	=	6	0	0
6s.×£2=12s.	=	0	12	0
8d.×£2=16d.	=	0	1	4
£3×5s.=15s.	=	0	15	0
6s.×5s.=$\frac{30}{20}$s.	=	0	1	6
8d.×5s.=$\frac{40}{20}$d.	=	0	0	2
£3×7d.=21d.	=	0	1	9
6s.×7d.=$\frac{42}{20}$d.	=	0	0	$2\frac{1}{10}$
8d.×7d.=$\frac{56}{240}$d.	=	0	0	$0\frac{7}{30}$
Ans.		7	11	$11\frac{1}{3}$

SINGLE RULE OF THREE DIRECT.

THE Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

If *more* require *more*, or *less* require *less*, the question belongs to the Rule of Three Direct.

But if *more* require *less*, or *less* require *more*, it belongs to the Rule of Three Inverse.*

RULE.†

1. State the question, by making that number, which asks the question, the third term, or putting it in the third place: That, which is of the same name or quality as the demand, the first term; and that which is of the same name or quality with the answer required, the second term.

* *More* requiring *more*, is when the third term is greater than the first, and requires the fourth term to be greater than the second. And *less* requiring *less*, is, when the third time is less than the first, and requires the fourth term to be less than the second.

Also, *more* requiring *less*, is when the third term is greater than the first, and requires the fourth term to be less than the second. And *less* requiring *more* is when the third term is less than the first, and requires the fourth term to be greater than the second.

† This Rule, on account of its great and extensive usefulness, is sometimes called the *Golden Rule of Proportion:* for, on a proper application of it and the preceding rules, the whole business of Arithmetick, as well as every mathematical inquiry, depends.—The rule itself is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause. Thus, the quantity of goods, bought, is in proportion to the money laid out;—the space, gone over by an uniform motion, is in proportion to the time, &c.

It has been shown in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole; and in Division, that the price of the whole divided by the quantity is the price of one: Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain, that the answer, found by this rule, will be the same as that found by multiplication of money, and where one is the last term of the proportion, it will be the same as that, found by Division of Money.

In like manner, if the first term be any number whatever, it is plain that the product of the second and third terms will be greater than the true answer required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit; consequently this product, divided by the first term, will give the true answer required.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and compound Division.

2. Multiply the second and third numbers together; divide the product by the first, and the quotient will be the answer to the question, which (as also the remainder) will be in the same denomination you left the second term

2. If the first term, and either the second or third can be divided by any number without a remainder, let them be divided, and the quotient used instead of them.

The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.

1. Divide the second term by the first; multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first; multiply the quotient into the second, and the product will be the answer.

3 Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient and the last quotient will be the answer.

Note. The term which asks or moves the question, has generally some words like these before it, viz What will? What cost? How many? How far? How long? How much? &c.

OBSERVATIONS.

1. The fourth number is always found in the name in which the second is given, or reduced to; which, if it be not the highest denomination of its kind, reduce to the highest when it can be done.

2. When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name to which the second is reduced, which reduce back to the highest possible.

3. If the first and third be of different names, or one or both of divers denominations, reduce them both to the lowest denomination mentioned in either.

4. When the product of the second and third is divided by the first; if there be a remainder after the division, and the quotient be not the least denomination of its kind; then multiply the remainder by that number, which one of the same denomination with the quotient contains of the next less, and divide this product again by the first number; and thus proceed, till the least denomination be found, or till nothing remain.

5. If the first number be greater than the product of the second and third; then bring the second to a lower denomination.

6. When any numbers of barrel, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the lowest name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it most generally happens) put the separate content of each properly under one another, then add them together, and you will have *the whole* quantity.

in, and which may be brought into any other denomination required.

Two or more statings, are sometimes necessary, which may always be known from the nature of the question.

The method of Proof is by inverting the question.

Examples.

1. If 6lb of sugar cost { 9s. / D.1 50c. } what will 30lb cost at the same rate?

lb	s.	lb		lb	D.	c.	lb
As 6	: 9	:: 30	: the Ans.	As 6	: 1	50 :: 30	: the Ans.
	9					30	
	6)270				6)45·00		
	45s. = £2 5s. Ans.				D 7·50 Ans.		

Here the first clause (if 6lb of sugar cost 9s. or D.1 50c.) supposes the rate; then follows the question: What will 30lb cost? 30lb. which moves the question, is the 3d term. 6lb the same kind, is the 1st, and 9s. (or D.1 50c.) the 2d.

Again. By inverting the order of the question, it will be,

2. If { 9s. / D.1 50c. } buy 6lb of sugar, how much will { £2 5s. / D.7 50c. } buy at that rate?

s.	lb	s.	
As 9	: 6	:: 45	: the answer.
		6	
	9)270		
	30lb Ans.		

D.	c.	lb.	D.	c.	
As 1	50	: 6	:: 7	50	: the answer.
				6	
			15)45·00(30lb answer.		
			45		
			0		

Again, 3. If 30lb. of sugar be worth $\left\{\begin{matrix}£2\ 5s.\\ D.7\ 50c.\end{matrix}\right\}$ how much may I buy for $\left\{\begin{matrix}9s.\\ D.1\ 50c.\end{matrix}\right\}$?

s. lb. s.
As 45 : 30 :: 9 : the answer.
9

45)270(6lb. the answer.
270

D. lb. D.
As 7·50 : 30 :: 1·50 : the answer.
30

7·5)45·00(6lb. answer.
45 0

Again, 4. Suppose $\left\{\begin{matrix}£2\ 5s.\\ D.7\ 50c.\end{matrix}\right\}$ will buy 30lb. of sugar, what will 6lb. of the same sugar cost?

lb. s. lb.
As 30 : 45 :: 6 : the answer.
6

3|0)27|0

9s. answer.

lb. D. lb.
As 30 : 7·50 :: 6 : the ans,
6

3|0)45·0|0

D.1·50 answer.

N. B. The three last questions are only the first *varied*, being put merely to show how any question, in this Rule, may be inverted.

5. If 5yds. cloth cost $\left\{\begin{matrix}£1\ 10s.\\ D.5\end{matrix}\right\}$ what will 20yds. ditto come to?

yds. s. yds.
As 5 : 30 :: 20
30 ÷ 5 = 6

2|0)12|0s.=£6 the answer.

Here I divide the 2d term by the 1st, and multiply the quotient into the 3d for the answer.

Again, 6. yds. s. yds.
As 5 : 30 :: 20
20÷5 = 4

120s.=£6 answer.

Here I divide the 3d term by the 1st, and multiply the quotient into the 2d, for the answer.

These operations would be, perhaps, still more apparent, if performed in Federal Money. Thus:

yds. D. yds.
As 5 : 5 :: 20
5 ÷ 5 = 1

D.20 answer.

yds. D. yds.
As 5 : 5 :: 20
20÷5 = 4

D.20 answer.

7. If 20 yards cost D.120, how many yards may I have for D.30?

D. yds. D.
As 120 : 20 :: 30

120÷20=6 quotient, and 30÷6=5 yards, answer.

Here I divide the 1st term by the 2d, and then, the 3d term by the quotient for the answer.

Again, 8. D. yds. D.
As 120 : 20 :: 30

120÷30=4 quotient, and 20÷4=5 yards, answer.

Here I divide the 1st term by the 3d, and then the 2d. term by that quotient for the answer.

9. If 1cwt. of tobacco cost £5 12 9½, what will 8cwt. ditto cost?

cwt. £ s. d. cwt.
1 : 5 12 9½ :: 8
8

Ans. £45 2 4

Here there is no need of reducing the middle term, because it can be performed by compound multiplication, the first term being an unit.

10. If 8cwt. of tobacco cost £45 2 4, what is that per cwt?

	£	s.	d.
8)	45	2	4
Ans.	5	12	9½

Here there is no need of reducing the middle term, because it may be performed by compound division only, the third term being an unit.

1. If 9cwt. 3qrs. sugar cost {£27 17s. 6d. / D.92 91c. 7m.} what will 2cwt. 1qr. 11lb. cost?

Cwt.	qrs.	£	s.	d.	Cwt.	qr.	lb.
9	3	27	17	6	2	1	11
4		20			4		
39		557			9		
28		12			28		
312		6690			73		
78					19		
1092					263		

lb. d. lb.
As 1092 : 6690 :: 263 : the answer.

263
2007
4014
1338
12
1092)1759470(1611
1092
2|0)13|4 3d.
£6 14s. 3d. answer.
6674
6552
1227
1092
1350
1092
258
4
1092)1032(0qr.

NOTE 1. If you look at the stating, you will see that the first and third terms are of the same kind, but of different denominations, and therefore are reduced to the same name or denomination, and that the demand of the questions lies on the 3d term.

2. That the middle term, being given in pounds, shillings and pence, is reduced to pence. But

3. If the second term were in federal money, it would be sufficient to proceed according to decimals. Thus:

	lb.		D.		lb.	
As	1092	:	92·917	::	263	: the answer.

263

278751
557502
185834

D.c.m.

1092)24437·171(22·378+, answer.
2184
2597
2184
4131
3276
8557
7644
9131
8736
395

12. If 57 yards cost { £69 / D.230 } what will 9 yards cost at that rate?

	yds.		£.		yds.
As	57	:	69	::	9

9

57)621(10 answer
57

51
20

1020 Carried over

Brought over. 57)1020(17s.
57
450
399
51
12
57)612(10d.
57
42
4
57)168(2$\frac{54}{57}$qrs.
114

Here, all the terms being whole numbers, there is no need of reducing the middle one until after stating.

The same in Federal maney would stand thus:

yds. D. yds.
As 57 : 230 :: 9 : the answer.
9

57)2070(36D. 31c. 5$\frac{15}{57}$m.
171
360
342
180
171
90
57
330
285
45 15
57 19

13. If my income be 109 guineas per annum, I desire to know what I may spend per day, so that I may lay up 45l. at the year's end? Ans. 5s. $10\frac{3}{4}$d. $\frac{1}{365}$ per day.

NOTE 1. You must substract 45l. from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus:

D. Guin. £ D. s. d. qr.
As 365 : 109—45 :: 1 : 5 10 $3\frac{1}{365}$ the Ans.

14. If my salary be 43l. 12s. 5d. per annum, what does it amount to per week? Ans. 16s. $9\frac{1}{52}$d.

The Stating W. £ s. d. W.
As 52 : 43 12 5 :: 1 : the answer.

NOTE. As there are 52 weeks and 1 day in a year, you will get the true answer to the above question by the following ratio.

D. £ s. d. D.
As 365 : 43 12 5 :: 7 : 16s. $8\frac{3}{73}$d.

15. Suppose my income to be 16s. $8\frac{3}{73}$d. per week, what is it per annum? Ans. 43l. 12s. 5d.

The Stating.

D. s. d. D.
As 7 : 16 $8\frac{3}{73}$:: 365 : 43l. 12s. 5d. Ans.

16. If I am to pay 1s. 7d. per week for pasturing a cow; what must I give per week for 37 cows?

C. s. d. C.
As 1 : 1 7 :: 37 : 2l. 18s. 7d. Ans.

17. How many yards of cloth may be bought for D.195 75c. of which $9\frac{1}{2}$yds. cost 11dol. 2c.?

D. c. yds. D. c. yds. qrs.
As 11 02 : $9\frac{1}{2}$:: 195 75 : 168 3 Ans.

18. If I buy 57 yards of cloth for 49 guineas; what did it cost per ell English?

yds. guin. yds.
As 57 : 49 :: $1\frac{1}{4}$: £1. 10s. $1\frac{13}{228}$d. Ans.

19. A merchant, failing in trade, owes in all £3475, and has in money and effects but £2316 13 4: Now supposing his effects are delivered up, pray what will each creditor receive on the pound?

£ £ s. d. £
As 3475 : 2316 13 4 :: 1 : £0 13s. 4d. Ans

20. A owes B. 3475l. but B compounds with him for 13s. 4d. on the pound: Pray what must he receive for his debt.

£ s. d. £
As 1 : 13 4 :: 3475 : 2316l. 13s. 4d. Ans.

21. If the distance from Newburyport to York be 31 miles; I demand how many times a wheel, whose circumference is 15½ feet, will turn round in performing the journey.

Feet. Cir. M. Cir.
As 15½ : 1 :: 31 : 10560 times, Answer.

22. Bought 9 chests of tea, each weighing 3 cwt. 2qrs. 21lb. at 4l. 9s. per cwt.; what came they to?

Cwt. £ s. C. qr. lb.
As 1 : 4 9 :: 3 2 21×9 : 147l. 13s. 8½d.

23. What will 37½ gross of buttons come to at 13 cents per dozen?

Doz. c. Gross. D. c.
As 1 : 13 :: 37½ : 58 50 Ans.

24. A farm, containing 125ac. 3r. 27p. is rented at Dol 1 50c. per acre; what is the yearly rent of that farm?

ac. D. c. ac. r. p. D. c. m.
As 1 : 11 50 :: 125 3 27 : 1447 6 5½ Ans.

25. If a ship cost 537l. what are ⅜ of her worth?

Eight. £ Eight.
As 8 : 537 :: 3 : £201 7s. 6d. Ans.

26. If $\frac{7}{16}$ of a ship cost 1163D. what is the whole worth?

Sixt. D. Sixt. D. c. m.
As 7 : 1163 :: 16 : 2658 28 5 Ans.

27. Bought a cask of wine at 76c. 6m. per gallon, for 125 dollars; how much did it contain?

Ans. 163gal. 1qt. $1\frac{3}{17}$pt.

28. What comes the insurance of 537l. 15s. to, at 4½l. per centum?

£ £ £ s. £ s. d.
As 100 : 4½ :: 537 15 : 24 3 11½ $\frac{8}{10}=\frac{4}{5}$ Ans.

29. What comes the commissions of 785l. to, at 3½ guineas per cent? Ans. 38l. 9s. 3½d. $\frac{4}{10}$qr.

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards: each package contained 8 parcels, each parcel 12 pieces, and each piece, 20 yards: how many dollars came the whole to, and how many per yard?

Yds. guin. pack. D.
As 7 : 3 :: 9 : 34560 Ans. for the whole cost.
Yds. guin. yd. D.
As 7 : 3 :: 1 : 2 Ans. per yard.

31. A merchant bought 49 tuns of wine for D.910; freight cost D.90; duties, D.40; cellar, D.31 67c.; other charges D.50, and he would gain D.185 by the bargain; what must I give him for 23 tuns?

Tuns. D. D. D. D. c. D. Tuns. D.
As 49 : 910+90+40+37 67+185 :: 23 : 613 33c. Ans.

32. If D.100 gain D.6 in a year, what will D.475 gain in that time? Ans. D.28 50c.

33. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles?

H. D. M. M. M.
As 24 : 360×51 :: 1 : 12¾ Ans.

34. Shipt for the West Indies 225 quintals of fish, at 15s. 6d. per quintal; 37000 feet of boards, at 8⅓ doll. per 1000; 12000 shingles, at ½guinea per 1000; 19000 hoops, at 1¼ doll. per 1000, and 53 half joes; and in return, I have had 3000gals. of rum, at 1s. 3d. per gallon; 2700 gallons of molasses at 5½d. per gallon; 1500lb. of coffee, at 8½d. per lb. and 19cwt. of sugar, at 12s. 3d. per cwt. and my charges on the voyage were 37l. 12s.: Pray, did I gain or lose, and how much, by the voyage?

Ans. lost £134 9 9.

35. If a staff, 4 feet long, cast a shade (on level ground) 7 feet; what is the height of that steeple, whose shade, at the same time, measures 198 feet?

F. sh. F. hei. F. sh. F. hei.
As 7 : 4 :: 198 : 113¼ Ans.

***36. Suppose a tax of D.755 be laid on a town, and the inventory of all the estates in the town amounts to 9345 dollars, what must A pay, whose estate is D.149?**

D.	D.	D.	D. c. m.
As 9345	: 755	:: 149	: 12 12 7 Ans.

* It may not be amiss to show the general method of assessing town or parish taxes. First, then, an inventory of the value of all the estates, both real and personal, and the number of polls, for which each person is rateable, must be taken in separate columns; The most concise way is then to make the total value of the inventory the first term, the tax to be assessed, the second, and D.1 the third, and the quotient will show the value on the dollar: 2dly, Make a table, by multiplying the value on the dollar by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, or otherwise, before stating to find the value on the dollar, you must deduct the sum of the average tax from the whole sum to be assessed: for which average, you must have a separate column, as well as for the real and personal estates.

EXAMPLE.

Suppose the General Court should grant a tax of D.500000, of which the town of Newburyport is to pay D 5312 50c. and, of which the polls being 1550, are to pay D.1 25c. each:—The town's inventory amounts to D.450000; what will it be on the dollar, and what is A's tax, whose estate, (as by the inventory) is as follows, viz. real D.1376, personal D.1149, and he has 3 polls?

	Pol.	D. c.	Pol.	D. c.
First, As 1	: 1 25	:: 1550	: 1937 50	the average part of the tax to be deducted from D.5312 50c. and there will remain D.3375.

	D.	D.	D.	
Secondly, As 45000	: 3375	:: 1	: 7½m.	on the dollar.

TABLE.

D.	D. c. m.	D.	D. c. m.	D.	D. c.
1 is	0 0 7½	20 is	0 15 0	200 is	1 50
2 —	0 1 5	30 —	0 22 5	300 —	2 25
3 —	0 2 2½	40 —	0 30 0	400 —	3 00
4 —	0 3 0	50 —	0 37 5	500 —	3 75
5 —	0 3 7½	60 —	0 45 0	600 —	4 50
6 —	0 4 5	70 —	0 52 5	700 —	5 25
7 —	0 5 2½	80 —	0 60 0	800 —	6 00
8 —	0 6 0	90 —	0 67 5	900 —	6 75
9 —	0 6 7½	100 —	0 75 0	1000 —	7 50
10 —	0 7 5				

Now, to find what A's rate will be,

37. If 52 gallons of water in one hour, fall into a cistern containing 230 gallons, and by a pipe in the cistern 35 gallons run out in an hour; in what time will it be filled.

Gal. gal. h. gal. h.
As 50—35 : 1 :: 230 : 15⅓ Ans.

38. A butcher went with £416, to buy cattle: Oxen, at £22 each, cows at £4, steers at £3 10s. and calves at £2 10s. and of each a like number; how many of each could he purchase with that sum?

£ £ £ s. £ s. each. £ each.
As 22+4+3 10+2 10 : 1 :: 416 : 13 Ans.

39. Said Harry to Dick, my purse and money are worth 3½ guineas, but the money is worth eleven times as much as the purse; pray how much money is there in it?

Guin. s. d.
As 12 : 1 :: 3½ : 7 7
Then £4 11s.—7s. 7d.=£4 3s. 5d. Ans.

40. How many dozen pair of gloves, at 13 groats per pair, may I have for 125 dollars?

Gr. pr. dol. doz. pr.
As 13 : 1 :: 125 : 14 5$\frac{4}{13}$ Ans.

41. There is a cistern, having four cocks; the first will empty it in ten minutes; the second in 20 minutes; the third in 40, and the fourth in 80 minutes: in what time will all four running together, empty it?

His real estate being D.1376, I find, by the table,

That	D.1000	is	D.7 50c.
That	D. 300	is	2 25
That	70	is	52 5m.
And that	6	is	4 5
For his real estate,			D.10 32

In like manner, I find his tax for personal estate to be } D.8 61 7½ } +D.10 32 = D.22
His three polls, at D.1 25c. each are 3 75 }
68c. 7½m. or D.22 69c. Ans.

Real.			Personal.			Polls.			Total.		
D.	c.	m.	D.	c.	m.	D.	c.	m.	D.	c.	m.
10	32	0	8	61	7½	3	75		22	68	7½

$$\text{As} \left\{\begin{matrix}10\\20\\40\\80\end{matrix}\right\} \text{ Cist. } : 1 :: 60 : \text{ Min. } \left\{\begin{matrix}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{matrix}\right\}$$

$11\frac{1}{4}$

Cist. Min. Cist. Min.

As $11\frac{1}{4} : 60 :: 1 : 5\frac{1}{3}$ Ans.

42. A and B depart from the same place, and travel the same road: but A goes 5 days before B at the rate of 20 miles per day; B follows at the rate of 25 miles per day: In what time and distance will he overtake A?

M. M. D. M. D. D. D. M. D. M.

As 25—20 : 1 :: 20×5 : 20. And, As 1 : 25 :: 20 : 500

43. Sold a cargo of flaxseed in Ireland, for £1795 10s. Irish money; what does that amount to, in Massachusetts currency, £81 5s. Irish being equal to £100 Massachusetts?

Irish. Mass. Irish. Mass.

As £81¼ : £100 :: £1795½ : £2209 16s. 11d. Ans.

Or, As £13 : £1795½ :: £16 : £2209 16s. 11d. as before, because £13 Irish are equal to £16 Massachusetts.

44. My correspondent in Maryland purchased a cargo of flour for me, for 437l. that currency; how much Massachusetts money must I remit him, 125l. Maryland being equal to 100l. Massachusetts, or 5 Mar.=4 Mass.?

Ans. 349l. 12s. Mass.

45. A bill of exchange was accepted at Newburyport for the payment of 345l. 10s. for the like value delivered in New-York, at 133⅓l. New-York currency for 100l. Massachusetts ditto; how much money was paid in New-York?

Mass. N. Y. Mass. N. Y.

As 75l. : 100l. :: 345l. 10s. : 460l. 13s. 4d. Ans.

46. When the exchange from Massachusetts to Georgia is 83⅓l. Georgia per 100l. Massachusetts, how much Massachusetts money must be paid in Boston to balance 457l. Georgia currency? Ans. 548l. 8s. Mass.

47. A merchant delivered at Boston, 320l. Massachusetts currency, to receive 400l. in Philadelphia; what was the Massachusetts pound valued at?

Ans. £1 5s. Phil.

48. If I draw a bill of exchange for 537l. 10s. 6d. Massachusetts, to be paid in Ireland, at $123\frac{1}{13}$l. Massachusetts, per 100l. Irish; for how much Irish money must I draw the bill? Ans. 436l. 14s. $9\frac{1}{2}$d. Irish.

49. Suppose a bill is drawn in Ireland, and payable in Boston, for 673l. 12s 6d. Irish; how much Massachusetts money comes it to, the exchange at $81\frac{1}{4}$l. Irish, per 100l. Massachusetts? Ans. 829l. 1s. $6\frac{6}{13}$d. Mass.

50. If I give D.1 75c. for 3 yards, how many yards may I have for D.180? Ans. 308yds. 2qrs. $1\frac{1}{7}$n.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.*

Having made the necessary preparations, as directed in Multiplication and Division of Vulgar Fractions, state your question as in whole numbers, and invert the first term of the proportion; then multiply the three terms continually together, and the product will be the answer.

EXAMPLES.

1. If $\frac{4}{5}$ of a yard cost $\frac{5}{7}$ of a £. what will $\frac{9}{13}$ of an Ell English cost?

$$\frac{4}{5}\text{yd.} = \frac{4}{5} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{5\times4\times1}{8\times1\times5} = \frac{1}{2} \text{ ell Eng.}$$

E. Eng. £ E. Eng.

As $\frac{1}{2}$: $\frac{5}{7}$:: $\frac{9}{13}$: $\frac{2}{1}\times\frac{5}{7}\times\frac{9}{13} = \frac{2\times5\times9}{1\times7\times13} = \frac{90}{105}$l. = 17s. $1\frac{1}{2}$d.

$\frac{4}{7}$ answer.

2. If $\frac{2}{3}$yd. cost $\frac{7}{8}$dol. what will $40\frac{1}{2}$ yards come to?

Ans. D.59 6c. $2\frac{1}{2}$m.

3. A person having $\frac{4}{5}$ of a vessel, sells $\frac{3}{4}$ of his share for D.1080$\frac{2}{3}$; what is the whole vessel worth?

Ans. D.2026 25c.

4. A merchant sold $5\frac{1}{3}$ pieces of cloth, each containing $12\frac{2}{3}$yds. at $12\frac{3}{5}$c. per yard; what did the whole amount to?

Ans. D.8 $82\frac{4}{5}$c.

* This rule and the next, depend upon the same principle as the Rule of Three in whole numbers.

5. A buys of B 560¾l. bank stock, at 85¾l. per cent.; what comes it to? Ans. 480l. 7s. 6½d.

6. A merchant bought a number of bales of velvet, each containing 129$\frac{17}{17}$ yards, at the rate of 7 dollars for 5 yards, and sold them out at the rate of 11 dollars for 7 yards; and gained 200 dollars by the bargain; how many bales were there?

yds. Dol. yds. Dol.
As 7 : 11 :: 5 : 7$\frac{4}{7}$

Sold 5 yards for 7$\frac{4}{7}$ dollars.
Bought 5 yards for 7 dollars.

In 5 yards gained $\frac{4}{7}$ dollar.

Dol. yds. Dol. yds.
As $\frac{4}{7}$: 5 :: 200 : 1166$\frac{2}{3}$, and

yds. B. yds. B.
As 129$\frac{17}{17}$: $\frac{1}{4}$:: 1166$\frac{2}{3}$: 9 answer.

Although the method before laid down be universally applicable, yet there are other methods more ready and expeditious in some particular cases.

Rule I.

If the first and third term be fractions, and the second a whole number, reduce the first and third to one common denominator, then, rejecting the denominators, make the numerator of the first, the first term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{5}{7}$ yard cost 9s. what cost $\frac{7}{12}$ yard at that rate?

$\frac{5}{7}=\frac{15}{24}$ and $\frac{7}{12}=\frac{14}{24}$. Now, As 15 : 9s. :: 14 : 8s. 4$\frac{3}{4}$d. Ans.

Rule II.

If of the first and third terms, one be 1, and the other a fraction; put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land cost 12l. what will $\frac{5}{8}$ of an acre cost at that rate?

Den. £ Num. £ s.
As 8 : 12 :: 5 : 7 10 Ans.

Rule III.

If the second term be a fraction likewise, (that is, if all the terms be fractions) having reduced the first and third to one common denominator, multiply the numerator of the first term by the denominator of the second, for a divisor; and the numerator of the third by the numerator of the second, for a dividend; divide the last product by the first, and the quotient will be the answer.

If $\frac{1}{2}$ yard of cloth cost £$\frac{3}{4}$ what cost $\frac{7}{8}$ yard?

$\frac{1}{2}=\frac{4}{8}$, which reduces it to a common denominator; then

As 4 : $\frac{3}{4}$:: 7

4 3

16 16)21(1$\frac{5}{16}$l.=26s. 3d. Ans.

16

5

RULE OF THREE DIRECT IN DECIMALS.

Rule.

Having reduced your fractions to decimals, and stated your question as in whole numbers, multiply the second and third together; divide by the first, and the quotient will be the answer.

Examples.

1. If $\frac{5}{8}$ of a yard cost $\frac{7}{12}$ of a pound; what will 9$\frac{2}{3}$ yards come to?

$\frac{5}{8}$=·625, $\frac{7}{12}$=·583+, and $\frac{2}{3}$=·667—.

As ·625 : ·583 :: 9·667

·583

29001

77336

48335

·625)5·635861(9·017+=9l. 0s. 4d. Ans.

5625

1086

625

4611

4375

236

2. If 1oz. of silver cost 6s. 8d. what is the price of a bowl, which weighs 1lb. 7oz. 13gr. Ans. 6l. 6s. 10d.

3. If $9\frac{3}{4}$ yards cost D.11 25c. what will $\frac{1}{4}$ yard come to?
Ans 57c. $6\frac{2}{13}$m.

4. There is a cistern, which has 3 cocks, the first will empty it in $\frac{1}{4}$ of an hour, the second in $\frac{3}{4}$, and the third in $1\frac{1}{2}$ hour: In what time will it be emptied if all three run together?

As $\begin{cases} \cdot25 : 1 :: 1 : 4 \\ \cdot75 : 1 :: 1 : 1{\cdot}333+ \\ 1{\cdot}5 : 1 :: 1 : 0{\cdot}667- \end{cases}$

6 Cisterns

Cist. ho. Cist.
As 6 : 1 :: 1 : ·1667—=10min. Ans.

5. A conduit has a cock, which running into a cistern, will fill it in 12 minutes: This cistern has three cocks; the first will empty it in $1\frac{1}{4}$ of an hour, the second in $37\frac{1}{2}$ minutes, and the third in $\frac{1}{2}$ an hour: In what time will the cistern be filled, if all four run together?

As $\begin{cases} \cdot2 : 1 :: 1 : 5 \\ 1{\cdot}25 : 1 :: 1 : 0{\cdot}8 \\ \cdot625 : 1 :: 1 : 1{\cdot}6 \\ \cdot5 : 1 :: 1 : 2 \end{cases}$ the filling cock. emptying cocks.

4·4
5 cisterns filled in an hour.
4·4 ditto emptied in do.
·6 cistern difference.

Cist. h. Cist.
Then, as ·6 : 1 :: 1 : 1·67—=1h. 40m. Ans.

6. If 19 yards cost D.25·75, what will $435\frac{1}{2}$ yards come to?

yds. D. d. c. yds.
As 19 : 25· 7 5 :: 435·5
25·75

21775
30485
21775
8710

D. d. c. m.
19)11214·125(590· 2 1 $7\frac{2}{19}$ Ans.

7. If 345 yards of tape cost D.5· 1d. 7c. 5m. what will 1 yard cost?

yds. D. yds. c.m.
As 345 : 5·175 :: 1 : ·015 Ans.

8. If I give D.12 8d. 2c. 5m. for 675 tops, how many tops will 19 mills buy? Ans. 1 top.

RULE OF THREE INVERSE,

Or, RECIPROCAL PROPORTION,

TEACHES, by having three numbers given to find a fourth, which shall have the same proportion to the second, as the first has to the third.

Therefore, the greater the third term is, in respect to the first, the less will the fourth term be in respect to the second; or, the less the third term is in proportion to the first, the greater the fourth must be in proportion to the second; and this is called *reciprocal, inverted,* or *indirect* Proportion.

The principal difficulty that will embarrass the learner will be, to distinguish when the proportion is direct, and when indirect. This is done by an attentive consideration of the sense and tenour of the question proposed: For if thereby it appears that, when the third term of the stating is less than the first, the answer must be less than the second; or, when the third is greater than the first, the answer must be greater than the second; then the proportion is direct: But, if the third be less than the first, and yet the sense of the question requires the fourth to be greater than the second; or the third, being greater than the first, the answer must be less than the second, the proportion is inverse.

RULE.*

State and reduce the terms as in the Rule of Three Direct; then, multiply the first and second terms together,

* The reason of this rule may be explained from the principles of Compound Multiplication and Compound Division, in the same

and divide the product by the third: the quotient will be the answer in the same denomination as the middle term was reduced into.

If there be fractions in your question, they must be stated as before directed, and if they be vulgar, invert the third term: Then multiply the three terms continually together, and the product will be the answer.

Examples.

1. How much shalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{3}{4}$ yards of cloth which is $1\frac{1}{4}$ yard wide?

yd.	yds.	qrs.		qrs.	qrs.	qrs.
As $1\frac{1}{4}$	: $6\frac{3}{4}$	:: 3	As 5	: 27	:: 3	
4	4			5		
5	27			3)135		
				4)45		
				$11\frac{1}{4}$ yards, Ans.		

The same by Vulgar Fractions.

First, $1\frac{1}{4}=\frac{5}{4}$, $6\frac{3}{4}=\frac{27}{4}$, and 3 qrs.$=\frac{3}{4}$. Then,

As $\frac{5}{4}$: $\frac{27}{4}$:: $\frac{3}{4}$. And $\frac{5}{4}\times\frac{27}{4}\times\frac{4}{3}=\frac{5\times27\times4}{4\times4\times3}=\frac{540}{48}=\frac{45}{4}=11\frac{1}{4}$ yds. the answer.

manner as the direct rule.—*For example*, If 4 men can do a piece of work in 12 days, in what time will 8 men do it?

As 4 men : 12 days :: 8 men : $\frac{4\times12}{8}$ = 6 days, the Ans.

And here the product of the first and second terms, that is, 4 times 12, or 48, is evidently the time in which one man would perform the work. Therefore, 8 men will do it in one eighth part of the time, or 6 days.

The same by Decimal Fractions.

$1\frac{1}{4}=1{\cdot}25$, $6\frac{3}{4}=6{\cdot}75$, and 3qrs.$={\cdot}75$. Then

As 1·25 : 6·75 :: ·75

1·25

3375
1350
675

·75)8·4375(11·25 yards, Answer
75

93
75

187
150

375
375

2. What length of board, $7\frac{1}{2}$ inches wide, will make a square foot?

In. br. in. len. in. br. in. len.
As 12 : 12 :: $7\frac{1}{2}$: $19\frac{1}{2}$ Ans.

3. How many yards of carpet, $2\frac{3}{4}$ feet wide, will cover a floor which is 18 feet long and 16 feet wide?

ft. ft. ft. ft. yds
As 16 : 18 :: $2\frac{3}{4}\times 3$: $34\frac{10}{11}$ Ans.

NOTE. I multiply $2\frac{3}{4}$ by 3, because 3 feet=1 yard.

4. Suppose I lend a friend £350 for 5 months, he promising the like kindness; but, when requested, can spare but £125; how long may I keep it to balance the favour?

£ Mo. £ Mo.
As 350 : 5 :: 125 : 14 Ans.

5. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain 9 months?

Mo. M. Mo. M. M.
As 5 : 450 :: 9 ; 250, and 450—250=200 men Ans.

6. If a man perform a journey in 15 days, when the day is 12 hours long, in how many days will he do it, when the day is but 10 hours? Ans. 18 days.

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide must it be when it is but 19 rods long, to make an acre? Ans. 8 rods 6ft. $11\frac{7}{19}$in.

8. If when wheat is D.1 per bushel, the two-penny loaf weigh 9·6oz. what ought it to weigh, when wheat is D.1 25c. per bushel? Ans. 7oz. 13pwt. 14·4gr.

9. If a piece of board be 30 inches in length, what breadth will make $1\frac{1}{2}$ square foot? Ans. 7·2 inches.

10. If 9 men can build a house in 5 months, by working 14 hours per day, in what time will the same number of men do it, when they work only 10 hours per day?

Ans. 7 months.

11. A wall, which was to be built 24 feet high, was raised 8 feet by 6 men, in 12 days: How many men must be employed to finish the wall in 4 days?

ft. m. ft. m.
As 8 : 6 :: 24—8 : 12 to finish it in 12 days; and
d. m. d. m.
As 12 : 12 :: 4 : 36 to finish in 4 days.

12. There is a cistern having a pipe which will empty it in 6 hours: how many pipes of the same capacity, will empty it in 20 minutes?

h. pi. mi. pi.
As 6 : 1 :: 20 : 18 Ans.

13. What number of men must be employed to finish in 9 days what 15 men would be 30 days about?

Ans. 50 men.

14. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

15. How much in length, that is $8\frac{1}{4}$ inches broad, will make a foot square? Ans. $16\frac{2}{3}\frac{4}{3}$ inches.

16. How much in length that is $13\frac{1}{3}$ poles in breadth, will make a square acre? Ans. $11\frac{52}{133}$ poles.

17. A regiment of soldiers, consisting of 745 men, is to be clothed, each suit to contain $3\frac{1}{4}$ yards of cloth, which

is $1\frac{3}{4}$ yard wide, and lined with shalloon $\frac{7}{8}$ yard wide; how many yards of shalloon will line them?

As $\overline{745 \times 3\frac{1}{2}}$: $1\frac{3}{4}$:: $\frac{7}{8}$: 4097½ yards, Ans.

18. If a suit of clothes can be made of $4\frac{1}{2}$ yards of cloth, $1\frac{3}{4}$ yard wide; how many yards of coating $\frac{7}{8}$ of a yard wide, will it require for the same person?

Ans. 6yds. 1qr. 3¼n.

COMPOUND PROPORTION,

OR DOUBLE RULE OF THREE,

TEACHES to resolve such questions as require two or more statings by simple proportion; and that, whether direct or inverse: It is composed (commonly) of five numbers to find a sixth, which if the proportion be direct, must bear such proportion to the 4th and 5th as the 3d bears to the 1st and 2d; but if inverse, the 6th number must bear such proportion to the 4th and 5th, as the 1st bears to the 2d and 3d.

RULE.

Always place the three conditional terms in this order: That number, which is the principal cause of gain, loss or action, possesses the first place; that, which denotes the space of time, distance of place, rate, medium or mean of action, the second; and that, which is the gain, loss or action, the third: This being done, place the other two terms which move the question, under those of the same name, and if the blank place, or term sought, fall under the third place, then the question is in direct proportion: therefore,

Rule 1.—Multiply the three last terms together for a dividend, and the two first for a divisor.—But if the blank fall under the first or second place; then, the proportion is inverse; therefore,

Rule 2.—Multiply the first, second, and last terms together for a dividend, and the other two for a divisor, and the quotient will be the answer.

EXAMPLES.

1. If D.100 gain D.6 in a year, what will D.400 gain in 9 months?

D.P. Mo. D. Int.

100 : 12 :: 6 } Terms in the supposition, or conditional terms.

400 : 9 Terms which move the question.

Here, the blank falling under the third place, the question is in direct proportion, and the answer must be found by the first rule: therefore,

400× 9×6=21600 For the dividend, and,
100×12 =1200 For the divisor.

See the work at large.

D.Pr. Mo. D.Int.
100 : 12 :: 6
400 : 9

100 3600
12 6

12|00)216|00(18D. Ans.
12
96
96

2. If D.100 will gain D.6 in a year, in what time will D.400 gain D.18?

D. Mo. D.
100 : 12 :: 6 Terms in the supposition.
400 : :: 18 Terms which move the question.

Here the blank falling under the 2d place, the question is in reciprocal or inverse Proportion, and the answer must be sought by the second rule: therefore,

100×12×18=21600 For the dividend.
400× = 2400 For the divisor.

D. Pr.		Mo.		D. Int.
100	:	12	::	6
400	:		::	18
6				12
2400				216
				100

24|00)216|00(9 months, Ans.
216

3. What principal, at 6 per cent. per ann. will gain D.18 in 9 months?

Pr.		Mo.		Int.
100	:	12	::	6
		9	::	18
				12
		9		216
		6		100

54)21600(400 D. Ans.
216

Here the blank falling under the first place, the proportion is inverse, and the answer found by the second rule, as in the last example.

4. If D.400 gain D.18 in 9 months; what is the rate per cent. per annum?

Pr.		Mo.		Int.
400	:	9	::	18
100	:	12	::	D.6 Ans.

5. If 8 men spend £32 in 13 weeks; what will 24 men spend in 52 weeks? Ans. £384.

6. If the freight of 9hhds. of sugar, each weighing 12cwt. 20 leagues, cost D.50; what must be paid for the freight of 50 tierces ditto, each weighing 2½cwt. 100 leagues?

Ans. D.289 35c. 1⅓m

7. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick?

m. da. ft.

6 : 16 :: $\frac{20\times6\times4}{200\times8\times6}$

24 : :: 80 days, Ans.

COMPARISON OE WEIGHTS AND MEASURES.

EXAMPLES.

1. If 78 pence Massachusetts be worth 1 French crown, how many Massachusetts pence are worth 320 French crowns?

F.cr. d. F.cr.

As 1 : 78 :: 320

78

2560

2240

24960 Ans.

2. If 24 yards at Boston make 16 ells at Paris, how many ells at Paris will make 128 yards at Boston?

Bost. Par. Bost. Par.

As 24yds. : 16ells :: 128yds. : 85$\frac{1}{3}$ells, Ans.

3. If 60lb. at Boston make 56lb. at Amsterdam, how many lb. at Boston will be equal to 350 at Amsterdam?

Ans. 375lb. Boston.

4. If 95lb. Flemish make 100lb. American, how many American lbs. are equal to 550lb. Flemish?

Ans. 578$\frac{13}{19}$lb. American.

CONJOINED PROPORTION

IS when the coins, weights or measures of several countries are compared in the same question; or, in other *words, it is joining* many proportions together, and by the *relation, which* several antecedents have to their con-

sequents, the proportions between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble, and it may be proved by as many statings in the Single Rule of Three, as the nature of the question may require.

CASE I.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

RULE.

Place the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right, and let the last number stand on the left hand; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

EXAMPLES.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouse, and 100 canes of Thoulouse=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many yards of America are equal to 379 ells of Hamburgh?

Antecedents.		Consequents.	Abridged.	
			Ant.	Con.
100 of America	=	100 of England.		
100 of England	=	50 of Thoulouse.	5	8
100 of Thoulouse	=	160 of Geneva.	379	
100 of Geneva	=	200 of Hamburgh.		
379 of Hamburgh?				

Therefore, $\frac{379 \times 5}{8} = 236\frac{7}{8}$ yds. of America=379 ells of Hamburgh.

ILLUSTRATION.

The two 100s of both sides cancel each other. Let the last cyphers of the three next antecedents and consequents *be cancelled,* which is dividing by 10. Then divide the

second antecedent and consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8. 10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2. Now, there being two left on each side, they cancel each other, and as there is no farther room for abridging by reason of that odd number 379, the operation is finished, and the answer found as before.

2. If 12lb. at Boston make 10lb. at Amsterdam, and 100lb. at Amsterdam 120lb. at Paris: How many lb. at Boston are equal to 80lb. at Paris? Ans. 80lb.

3. If 40lb. at Newburyport make 36 at Amsterdam, and 90lb. at Amsterdam make 116 at Dantzick: How many lb. at Newburyport are equal to 260lb. at Dantzick?

Ans. $224\frac{4}{29}$.

CASE II.

When it is required to find how many of the last sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the first.

Rule.

Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand; then multiply the first row for a divisor, and the second for a dividend.

Examples.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouse, and 100 canes of Thoulouse=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many ells of Hamburgh are equal to $236\frac{7}{8}$ yards of America?

Ante.	Conse.	Abridged.
100 Amer.	= 100 Eng.	Ant. Con.
100 Eng.	= 50 Thoul.	5 8
100 Thoul.	=160 Gen.	$236\frac{7}{8}$
100 Gen.	=200 Ham.	$236\frac{7}{8}\times 8$
$236\frac{7}{8}$	Amer.	$\frac{}{5}$ =379 Ham. Ans.

This needs no further illustration. The learner will readily see that this case being the reverse of the former, they are proofs to each other.

2. If 12lb. at Boston make 10lb. at Amsterdam, and 100lb. at Amsterdam 120lb. at Paris: How many at Paris are equal to 80lb. at Boston? Ans. 80lb.

3. If 40lb. at Newburyport make 36 at Amsterdam, and 90lb. at Amsterdam make 116 at Dantzick: How many lb. at Dantzick are equal to 244 at Newburyport?

Ans. 283$\frac{1}{25}$lb.

ARBITRATION OF EXCHANGES.

By this term is understood how to choose, or determine the best way of remitting money abroad with advantage; which is performed by conjoined proportion: Thus,

Suppose a merchant has effects at Amsterdam to the amount of 3530 dollars, which he can remit by way of Lisbon at 840 rees per dollar, and thence to Boston, at 8s. 1d. per milree (or 1000 rees): Or, by way of Nantz, at 5$\frac{4}{5}$ livres per dollar, and thence to Boston at 6s. 8d. per crown; it is required to arbitrate these exchanges, that is, to choose that which is most advantageous?

1 dollar at Amsterdam = 840 rees at Lisbon.
1000 rees at Lisbon = 97d. at Boston.
3530 dollars at Amsterdam.

$$\frac{840\times97\times3530}{1000\times1}=£1198\ 8s.\ 8\tfrac{4}{10}d.$$ by way of Lisbon.

1 dollar at Amsterdam = 5$\frac{4}{5}$ livres at Nantz.
6 livres at Nantz = 80 pence at Boston.
3530 dollars at Amsterdam.

$$\frac{5\frac{4}{5}\times80\times3530}{1\times6}=£1059$$ by way of Nantz.

Here it may be observed that the difference is £139 8s. 8$\frac{4}{10}$d. in favour of remitting by way of Lisbon rather than by Nantz, which depends on the course of exchange, at that *time: but the course* may vary so, that, in a short time by

way of Nantz, may be better; hence appears the necessity and advantage of an extensive correspondence, to acquire a thorough knowledge in the *courses* of exchange, to make this kind of remittance.

FELLOWSHIP.

THE Rules of Fellowship are those by which the accompts of several merchants or other persons, trading in partnership, are so adjusted, that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the joint stock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP

Is, when the stocks are employed for any certain equal time,

RULE.*

As the whole stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain or loss.

PROOF. Add all the particular shares of the gain or loss together, and if it be right, the sum will be equal to the whole gain or loss.

EXAMPLES.

1. Divide the number 360 into four such parts, which shall be to each other, as 3, 4, 5 and 6.

As $3+4+5+6 : 360 :: \left\{\begin{matrix} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{matrix}\right\}$ Answer.

360 Proof.

* That their gain or loss, in this Rule, is in proportion to their stocks, it is evident: For, as the times, in which the stocks are in trade, are equal, if I put in one half of the whole stock, I ought to have one half of the gain; if my part of the stock be one fourth, *my share of the gain* or loss ought to be one fourth also. And *generally the same ratio* that the whole stock has to the whole gain or *loss, must each person's* particular stock have to his respective *gain or loss.*

2. B, C, D, and E companied; B put in 145l.; C, 219l.; D, 378l. and E, 417l. with which they gained 569l.: What was the share of each?

Whole Stock.	Gain.	£	s.	d.	shares.
As 145+219+378+417 : 569 ::	145 :	71	3	8½	1044/1150 B's
	219 :	107	10	3¾	876/1150 C's
	378 :	185	11	6	552/1150 D's
	417 :	204	14	5¼	138/1150 E's
		569 — Proof.			

3. K, L, M, and N are concerned in a joint stock of D.1000; of which K's part is D.150; L's D.250; M's D.275, and N's 325; upon the adjustment of their accompts, they have lost D.337 50c. What is the loss of each?

	D.	c.
Ans. K	D.50	62½c.;
L	D.84	37½c.;
M	D.92	81¼c.
N	D.100	68¾c.

4. F, G and H freighted a ship with 68900 feet of boards: F put in 16520 feet; G 28750; and H the rest; but in a storm the captain threw overboard 26450 feet: How much must each sustain of the loss?

Ans. F 6341¾feet; G 11036¾, and H 9071¼do.

5. A ship, worth D.3000, being lost at sea, of which ¼ belonged to O, ⅛ to P, and the rest to Q: What loss will each sustain, supposing D.450 to have been insured upon her?

Ans. O's loss	D.312 50c.
P's	937 50
Q's	625

6. X and Y venturing equal sums of money, cleared by joint trade D.140; by agreement, as X did the business, he was to have 8 per cent. and Y was to have 5 per cent.:— What was X allowed for his trouble?

D.		D.		D.		D.	
As 8+5	:	140	::	8	:	86 8/13; and,	
As 8+5	:	140	::	5	:	53 1/13	

Ans. D.32 30c. 7 7/13m.

7. A bankrupt is indebted to R £120, to S £230, to T £340, and to U £450, and his whole estate amounts only to £560: How must it be divided among his creditors?

Ans. R £58 18 11¼; S £112 19 7¾: T £167 0 4; U £221 1 0¼.

8. X, Y, and Z put their money into a joint stock; X put in D.40; Y and Z together, D.170; they gained D.126, of which Y took D.42: What did X and Z gain, and Y and Z put in respectively?

As D.210 the whole stock: D.126 the whole gain :: D.40 X's stock: D.24 X's gain.

As D.24 X's gain : D.40 X's stock :: D.42 Y's gain : D.70 Y's stock. Then D.170—D.70=D.100 Z's stock; and whole gain D.126—D.66 X's and Y's gain= D.60 Z's gain.

9. E, F, G, and H companied; and gained a sum of money, of which E, F, and G took £120; F, G, and H, £180; G, H, and E, £160; and H, E, and F, £140; what distinct gain had each?

The sum of these 4 numbers is £600, and as each man's money is named 3 times, therefore $\frac{1}{3}$, viz. £200, is the whole gain. Therefore £200—£120 E's, F's, and G's gain=£80 H's gain; £200—£180 F's, G's, and H's gain= £20 E's gain; £200—£160 G's, H's and E's gain=£40 F's gain; and £200—£140 H's, E's and F's gain=£60 G's gain.

*DOUBLE FELLOWSHIP.**

Or *Fellowship with Time*, is occasioned by the shares of partners being continued unequal times.

Rule.

Multiply each man's stock, or share, by the time it was continued in trade. Then,

As the whole sum of the products, is to the whole gain or loss, so is each man's particular product, to his particular share of the gain or loss.

Examples.

1. A, B, and C hold a pasture in common, for which they pay 40l. per annum; A put in 9 oxen for 5 weeks;

* When times are equal, the shares of the gain or loss are evidently as the stocks, as in Single Fellowship; and when the stocks are equal, the shares are as the times; wherefore, when neither *are equal, the shares* must be their products.

B. 12 oxen for 7 weeks; and C, 8 oxen for 16 weeks.—What must each pay of the rent?

$9\times5=45$. $12\times7=84$, and $8\times16=128$; then $128+84+45=257$.

As 257 : 40 :: 45 As 257 : 40 :: 84 As 257 : 40 :: 128

45	84	40
200	160	257)5120(19
160	320	257
257)1800(7	257)3360(13	2550
1799	257	2313
1	790	237
20	771	20
257)20(0	19	257)4740(18
12	20	257
257)240(0	257)380(1	2170
4	257	2056
257)960(3	123	114
771	12	12
189	257)1476(5	257)1368(5
	1285	1285
	191	83
	4	4
	257)764(2	257)332(1
	514	257
	250	75

2. Four merchants traded in company; L put in 460 dollars for 5 months, M, D.600 for 7 months, N, D.960 for 8 months, and O, D.1200 for 9 months; but by misfortunes at sea, they lost D.750: What must each sustain of the loss?

Ans. { L D.60 77c. $7\frac{52}{257}$m. N D.233 38c. $7\frac{231}{257}$m.
{ M 127 63c. $3\frac{236}{257}$m. O 328 20c. $0\frac{228}{257}$m.

M

3. A, with a capital of 100l. began trade January 1st, 1787, and meeting with success in his business, he took in B as a partner on the first day of March following, with a capital of 150l. Three months after that, they admit C as a third partner, who brought into stock 180l. and after trading together until the first of January, 1788, they found there had been gained since A's commencing business, £177 13s. How must this be divided among the partners?

Ans. { A 53l. 16s. 8d. / B 67 5 10 / C 56 10 6

4. Two merchants entered into partnership for 18 months; K first put into stock D.400, and at the end of 8 months he put in 200 more; L, at first put in D.1100, and at the end of 4 months took out D.280. Now at the expiration of the time, they found they had gained D.1052:— What is each man's just share?

Ans. K, D.385 90c. L, D.666 10c.

5. N and O companied; N put in, the 1st of January, £150 but O could not put in any till the 1st of May: What did he then put in, to have an equal share with N at the year's end?

$$\text{As } 12 : 150 :: 8 : \frac{150 \times 12}{8} = £225 \text{ Ans.}$$

FELLOWSHIP BY DECIMALS.

Rule.

Divide the whole gain or loss, by the whole stock, or sum of all the products, as the case requires, and the quotient multiplied severally by each man's stock, or product, will give the gain or loss of each.

Examples.

1. H, I, and K companied; H put in £40 5s.; I, £80 10s.; and K, £161 with which they gained £120: What is each man's share of the gain?

H's stock = 40·25
I's do. = 80·5
K's do. = 161·

Sum total = 281·75)120·000000(·4259+

·4259	·4259	·4259
40·25	80·5	161
21295	21295	4259
8518	34072	25554
17036		4259
£17·142475	£34·28495	£68·5699
20	20	20
2·849500	5·69900	11·3980
12	12	12
10·194000	8·388	4·776
4	4	4
0·776000	1·552	3·104

Proof. H's gain £17 2s. 10d.+I's gain £34 5s. 8¼d.+K's gain £68 11s. 4¾d.=£119 19s. 11d.

2. L, M, and N companied; L put in D.400 for 8 months; M, D.300 for 9 months, and N, D.175 for 12 months; with which they gained D.720. Required the share of each?

L 400× 8 = 3200
M 300× 9 = 2700
N 175×12 = 2100

Sum of Products = 8000)720·(·09=quotient.

D.
3200 × ·09 = 288 = L's share,
2700 × ·09 = 243 = M's share,
2100 × ·09 = 189 = N's share,
} Ans.

3. An insolvent estate, amounting to D.633 60c. is indebted to A D.312 75c. to B D.297, to C D.50 25c. to D 25c. to E D.200, to F D.142 50c. and to G D.21 25c.; what proportion will each creditor receive?

$$\frac{633{\cdot}6}{312{\cdot}75+297+50{\cdot}25+{\cdot}25+200+142{\cdot}5+21{\cdot}25} = {\cdot}61875;$$ or,

61c. 8¾m. on a dollar. And,

D. c.		D c. m.	
312·75		193·51 $4\frac{1}{16}$	= A's proportion.
297·		183·76 $8\frac{3}{4}$	= B's do.
50·25		31·09 $2\frac{3}{16}$	= C's do.
·25	×·61875=	·15 $4\frac{11}{16}$	= D's do.
200·		123·75	= E's do.
142·50		88·17 $11\frac{7}{8}$	= F's do.
21·25		13·14 $8\frac{7}{16}$	= G's do.

Proof D.633·60

PRACTICE

IS a contraction of the Rule of Three Direct, when the first term happens to be an unit, or 1; and has its name from its daily use among merchants and tradesmen, being an easy and concise method of working most questions, which occur in trade and business.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

General Rule.

1. Suppose the price of the given quantity to be £1. or 1s. &c. then will the quantity itself be the answer at the supposed price.

2 Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients, belonging to each, will be the true answer required.

Example.

What is the value of 468 yards, at 2s. 9¼d. per yard?

	£468 s. d.	Answer at £1 s. d.
2s. 6d. is $\frac{1}{2}$ =	58 10 0	ditto at 0 2 6
3d. is $\frac{1}{12}$ =	5 17 0	ditto at 0 0 3
¼d. is $\frac{1}{12}$ =	0 9 9	ditto at 0 0 0¼
The full price=	£64 16 9	0 2 9¼

In this example it is plain, that the quantity 468 is the answer at £1; consequently, as 2s. 6d. is $\frac{1}{2}$ of a pound, $\frac{1}{2}$ part of that quantity, or £58 10s. is the price at 2s. 6d.; in

like manner, as 3d. is the $\frac{1}{16}$ part of 2s. 6d. so $\frac{1}{16}$ part of 58l. 10s. or 5l. 17s. is the answer at 3d.; and as 1qr. is $\frac{1}{12}$ of 3d. so $\frac{1}{12}$ of 5l. 17s. or 9s. 9d. is the answer at 1qr. Now as the sum of all these parts is equal to the whole price (2s. 9½d.) so the sum of the answers belonging to each price will be the answer at the full price required, and the same will be true in any example whatever.

General Rule.

To find the value of goods in Federal Money.—Multiply the price and quantity together; point off in the product for denominations lower than dollars, as many places as there are in the given price; or, if there be decimal places in the quantity, (or lower denominations previously reduced to decimals) according to multiplication of decimals.

Examples.

1. What cost 823 yards, at D.1 29c. per yard?

D. D.
823×1·29=1061·67 Ans.

2. What cost 56 yards 2qrs. at D.3·11 per yard:
56yds. 2qrs.=56·5yds.; and 56·5×3 11=D.175·715 Ans.

Before the questions, hereafter given, can be wrought, the following Tables must be perfectly gotten by heart.

TABLES.

Aliquot, or even Parts of Money.

Pts. of a s.		of a £
d.	s.	£
6	= ½	= 1/40
4	= ⅓	= 1/60
3	= ¼	= 1/80
2	= ⅙	= 1/120
1½	= 1/8	= 1/160
1	= 1/12	= 1/240
¾	= 1/16	= 1/320
½	= 1/24	= 1/480
¼	= 1/48	= 1/960

Parts of a £		
s.	d.	£
10	0	= ½

s.	d.		£
6	8	=	⅓
5	0	=	1/4
4	0	=	⅕
3	4	=	1/6
2	6	=	1/8
1	8	=	1/12
1	4	=	1/15
1	3	=	1/16
1	0	=	1/20
0	10	=	1/24
0	8	=	1/30
0	5	=	1/48
0	2½	=	1/96

Parts of a Dollar.		
c.		D.
50	=	½
33⅓	=	⅓
25	=	¼
20	=	⅕
16⅔	=	1/6
12½	=	⅛
8⅓	=	1/12
6¼	=	1/16
5	=	1/20
4	=	1/25
2	=	1/50

Aliquot, or even Parts of Weight.

Pts. of a Cwt. qrs. lb.	= Cwt.	Pts. of ½C. lb.	= ½Cwt.	Pts. of ¼C. lb.	= ¼Cwt.	Pts. of a Ton. Cwt.qr.	= T.
2 0	= ½	28	= ½	14	= ½	10 0	= ½
1 0	= ¼	14	= ¼	7	= ¼	5 0	= ¼
0 16	= ⅛	8	= ⅛	4	= ⅛	4 0	= ⅛
0 14	= ⅛	7	= ⅛	2	= 1/14	2 2	= ⅛
0 8	= 1/14	4	= 1/14			2 0	= 1/16
0 7	= 1/16					1 1	= 1/16
0 4	= 1/28					1 0	= 1/20

Another Table of aliquot Parts of Money.

Parts of a shill. d.	= s.	s. d.	= £	c.	= D.
10	= ⅚	13 4	= ⅔	80	= ⅘
9	= ¾	12 6	= ⅝	75	= ¾
8	= ⅔	12 0	= 3/5	70	= 7/10
7½	= ⅝	8 0	= 2/5	68¾	= 11/16
4⅙	= ⅓	7 6	= ⅜	66⅔	= ⅔
		6 0	= 3/10	60	= ⅗
				58½	= 7/12
				56¼	= 9/16
				43¾	= 7/16
				41⅔	= 5/12
				40	= ⅖
				37½	= ⅜
				31¼	= 5/16
				30	= 3/10
				18¾	= 3/16

Parts of a Pound. s. d.	= £	Parts of a Doll. c.	= D.
18 0	= 9/10	93¾	= 15/16
17 6	= 7/8	91⅔	= 11/12
16 8	= ⅘	90	= 9/10
16 0	= 4/5	87½	= 7/8
15 0	= ¾	83⅓	= ⅚
14 0	= 7/10	81¼	= 13/16

A TABLE OF DISCOUNT PER CENT.

£		s.	d.		£		s.	d.	
1¼	per cent. =	0	3	on the pound.	17½	per cent. =	3	6	on the pound.
2½	=	0	6		20	=	4	0	
3¾	=	0	9		22½	=	4	6	
5	=	1	0		25	=	5	0	
6¼	=	1	3		30	=	6	0	
7½	=	1	6		35	=	7	0	
8¾	=	1	9		40	=	8	0	
10	=	2	0		45	=	9	0	
12½	=	2	6		50	=	10	0	
15	=	3	0						

CASE I.

When the price of 1yd. lb. *&c. is an even part of one shilling:* Find the value of the given quantity at 1s. per yard, lb, &c.; then draw a line underneath and divide by *that even part*, and the quotient will be the answer in shillings, which must always be brought into pounds.

EXAMPLES.

1. What will 354½yds. cost at ¼d. per yard?

		s.	d.	
¼d.	1/48	354	6	value of 354½yds. at 1s. per yard.

Ans. £0 7 4½ value of 354½yds. at ¼d. per yard.

Or thus.

£.	s.	d.		s.	d.
2)17	14	6	=	354	6
6)2	4	3½			
7	4½	Ans. as before.			

Or divide by 8 and 6, thus,

8)354 6

6)44 3½

7 4½ Ans.

2. What will 759¾ yards come to at 3d. per yard?

3d.	¼	759	9 value at 1s. per yard.
2	0	18	9 11¼

Ans. £9 9 11¼ value at 3d. per yard.

Or thus.

		£	s.	d.	
3d.	¼	37	19	9	value at 1s. per yd.

Ans. £9 9 11¼ value of 759¾ yards, at 3d. per yard.

Questions. Yds.		Answers. £	s.	d.
3. 642	at ¼d. per yard.	0	13	4½
4. 567½	— 1¼d. ——	3	10	11¼
5. 475¼	— 4d. ——	7	18	5

CASE II.

When the price is pence, and no even part of a shilling: Find the value of the given quantity at 1s. per yard, divide

the pence into aliquot parts, for divisors, and the sum of the quotients arising from them will be the answer.

EXAMPLES.

1. What will 487½ yards come to, at 5d. per yard?

3d.	¼	24	7	6	value of 487½ yds. at 1s. per yard.
2d.	⅓	6	1	10½	value of ditto at 3d. per yard.
		4	1	3	value of ditto at 2d. per yard.

Ans. £10 3 1½ value of ditto at 5d. per yard.

3. 912½ yards, at 9d. per yard. Ans. £34 4s. 4½d.
4. 745¾ yards, at 11d. per yard. Ans. £34 3s. 7¼d.

CASE III.

When the price is between one and two shillings: Find the value of the quantity at 1s. per yard, &c. which value being divided by *those even parts,* which the pence are of 1s. and the quotient or quotients arising therefrom, added thereto, the sum will be the answer.

EXAMPLES.

1. What will 758½ yards come to at 1s. 9d. per yard?

6d.	½	37	18	6	value at 1s. per yard.
3d.	¼	18	19	3	value at 6d. per yard.
		9	9	7½	value at 3d. per yard.

Ans. £66 7 4½ value of 758½ yards, at 1s. 9d. per yard.

Questions.			Answers.		
2. 793 yards, at		12¾d.	£42	2	6¾
3. 847½	——	1s. 1d.	45	18	1½
4. 647¾	——	1s. 5d.	45	17	7¾
5. 752½	——	1s. 10d.	68	19	7

CASE IV.

When the price is an even part of a pound: Find the value of the given quantity, at one pound per yard, &c. then draw a line underneath, and divide by *that part;* the quotient will be the answer.

EXAMPLES.

1. What will 156¾ yards come to, at 3s. 4d. per yard?

s.	d.		£	s.	d.	
3.	4.	⅓	156	15	0	price at £1 per yard.

Ans. £26 2 6 price at 3s. 4d. per yard.

Questions. Yds.		s.	d.		Answers. £.	s.	d.
2. 516¾	at	1	0	per yard.	25	16	9
3. 624	—	1	3	——	39	0	0
4. 719½	—	1	4	——	47	19	4
5. 648	—	1	8	——	54	0	0

CASE V.

When the price wants an even part of a pound: First find the value of the given quantity at £1 per yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient therefrom; the remainder will be the answer.

EXAMPLES.

1. What will 167½ yds, cost, at 17s. 6d. per yard?

s.	d.		£	s.	d.	
2	6	⅛	167	10	0	value at £1 per yard.
			20	18	9	ditto at 2s. 6d. per yard.

Ans. £146 11 3 value at 17s. 6d. per yard.

Questions. Yds.		s.	d.		Answers. £	s.	d.
2. 347½	at	13	4	per yd.	231	13	4
3. 485¼	—	15	0	——	364	6	3
4. 614	—	16	0	——	491	4	0
5. 912¼	—	17	6	——	798	4	4½

CASE VI.

When the price is shillings, pence, and farthings, and not an even part of a pound: Multiply the given quantity by the shillings in the price of 1 yard, &c. and take parts of parts from the quantity for the pence, &c.; then add them together, and their sum will be the answer in shillings, &c. Or you may let the given quantity stand as pounds per yard,

&c. then draw a line underneath, and take parts of parts therefrom; which add together, and their sum will be the answer.

N. B. I advise the learner to work the following examples both ways, by which means he will be able to discover the most concise method of performing such questions in business, as may fall under this case.

EXAMPLES.

1. What will 248½ yards cost, at 7s. 6d. per yard?

| 6 | ½ | 248s. 6d. value of 248½ yards, at 1s. per yard.
7

1739 6 value of ditto at 7s. per yard.
124 3 value of ditto at 6d. per yard.

2|0)186|3 9

Ans. £93 3 9 value of ditto at 7s. 6d. per yard.

Or thus:

£ s. d.
| 6 | ½ | 12 8 6 value of 248½ yards, at 1s. per yd.
Mult. by 7

86 19 6 value of ditto at 7s. per yard.
6 4 3 value of ditto at 6d. per yard.

Ans. £93 3 9

By the latter part of this case.

| 5 0 | ¼ | 248 10 0 value of 248½ yards, at £1 per yd.
| 2 6 | ½ | 62 2 6 value of ditto at 5s. per yard.
31 1 3 value of ditto at 2s. 6d. per yard.

Ans. £93 3 9 value of ditto at 7s. 6d. per yard.

Questions.					Answers.		
Yds.		s.	d.		£	s.	d.
2. 68½	at	4	6	per yd.	15	8	3
3. 124	—	5	8	—	35	2	8
4. 146	—	14	9	—	107	13	6
5. 218½	—	12	6	—	136	11	3

CASE VII.

When the price of the yard, lb. *&c. is pounds, shillings, and pence*: First, multiply the quantity by the pounds, and if the shillings and pence be an even part of a pound, divide the given quantity by *that part*, and add the quotient to the product for the answer. But if they be not an even part of a pound, you must take parts of parts, and add them together as before. Or, reduce the pounds, and shillings into shillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their sum will be the answer in shillings, &c.

N. B. The learner should work the following questions both ways.

Examples.

1. What will 156 yards come to, at £3 6s. 8d. per yd?

s.	d.				
6	8	$\frac{1}{3}$	156	0	0 value at £1 per yard.
			3		
			468	0	0
			52		
Ans.			£520	0	0

Or thus:

4	$\frac{1}{3}$	156	value at 1s. per yard.
4	$\frac{1}{3}$	66	shillings in the price of 1 yard.
		936	
		936	
		10296	value at 3l. 6s. per yard
		52	
		52	
2	0	1040	0
		£520	0 0

Questions.					Answers.	
Yds.		£ s. d.			£	s. d.
2. 345½	at	6 5 0	per yd.		2159	7 6
3. 59¾	—	3 6 8	——		199	3 4
4. 75	—	5 3 4	——		387	10 0
5. 68	—	4 6 0	——		292	8 0

CASE VIII.

When the price of one hundred weight, &c. is of several denominations, and the quantity likewise: Multiply the price by the integers, and take parts for the rest from the price of an integer; which, added together, will be the answer.

Examples.

1. What will 9cwt. 3qrs. 14lb. of sugar come to, at 4l. 17s. 4d. per cwt.?

qrs.	lb.		£	s.	d.	
2	0	½	4	17	4	price of 1cwt.
1	0	¼			9	
0	14	½				

				Cwt.	qrs.	lb.
43	16	0	price of	9	0	0
2	8	8	——	0	2	9
1	4	4	——	0	1	0
0	12	2	——	0	0	14
Ans. £48	1	2	price of	9	3	14

Questions.					Answers.	
cwt. qr. lb.		£ s. d.			£.	s. d.
2. 8 1 16	at	5 17 9	per cwt.		49	8 $2\frac{3}{4}$
3. 7 3 19	—	7 12 8	——		60	9 $0\frac{1}{2}$
4. 12 1 24	—	3 18 10	——		49	2 $7\frac{1}{4}$
5. 16 2 17	—	2 15 11	——		46	11 1

CASE IX.

When the price is at any of the rates in the second Practice Table of aliquot parts: Multiply the given quantity by the numerator, and divide that product by the denominator; if the price be pence, the quotient will be the answer in shillings; if shillings, the answer will be pounds.

EXAMPLES.

1. What will 379 yards, at $4\frac{1}{2}$d. per yard, come to?

379
3
8)1137
2|0)14|2 $1\frac{1}{2}$
Ans. £7 2 $1\frac{1}{2}$

2. What will 149 yards, at 6s. per yard, come to?

149
3
1|0)44|7
Ans. £44 14

Questions.					Answers.		
		s.	d.		£	s.	d.
3.	127 yards, at	0	$7\frac{1}{2}$	per yard.	3	19	$4\frac{1}{2}$
4.	249 ditto	7	6	——	93	7	6
5.	357 ditto	12	6	——	223	2	6

CASE X.

To find the value of goods sold by particular quantities: viz. I. By the score. II. Round timber. III. By 5 score to the hundred. IV. By 112 to the hundred. V. By 6 score to the hundred. VI. By the great gross. VII. By the thousand.

I. *To find the value of Goods sold by the Score.*

The price of 1 is given to find the price of 1 score.

If the given price be shillings and pence, or only pence, divide the given price, in pence, by 12. The quotient will be the answer in pounds, and the remainder will be so many times 1s. 8d.

EXAMPLES.

1. At 9d. each, what is that per score?

12)9d.(·75=£0 15 0 Ans.
Or by inverting the question.
1 score=20=1s. 8d.
9
15s. 0

2. At 4s. 9d. each, what is that per score?

4s. 9d.
12
12)57d.
£4 15s. Ans.

It may be remarked, that when the price is shillings and pence, the answer will be just so many pounds as there are shillings, and so many times 1s. 8d. as there are pence. If farthings are given, for 1qr. reckon 5d. for 2qrs. 10d. and for 3qrs. 1s. 3d.

TABLE of Aliquot Parts. 20 the Integer.

$2 = \frac{1}{10}$	$6 = \frac{3}{10}$	$12 = \frac{6}{10}$	$16 = \frac{8}{10}$
$4 = \frac{1}{5}$	$8 = \frac{4}{10}$	$14 = \frac{7}{10}$	$18 = \frac{9}{10}$
$5 = \frac{1}{4}$	$10 = \frac{1}{2}$	$15 = \frac{3}{4}$	

3. What cost 7, at 2s. 9d. per score?

		s.	d.
5	$\frac{1}{4}$	2	9
2	$\frac{1}{10}$		
		0	$8\frac{1}{4}$
		0	$3\frac{1}{4}$
7	=	0	$11\frac{1}{2}$

4. What cost 17, at 19s. 10d. per score?

		s.	d.
10	$\frac{1}{2}$	19	10
5	$\frac{1}{4}$		
2	$\frac{1}{10}$	9	11
		4	$11\frac{1}{2}$
		1	$11\frac{3}{4}$
17	=	16	$10\frac{1}{4}$

II. *Round Timber.*

Forty feet make a load or ton of round timber.

If the given price of a foot be shillings,

RULE.

Multiply the given price by 2, and the product will be the answer in pounds.

5. What cost a ton, at 3s. per foot? 3s×2=6l. Ans.

6. What cost a ton, at 9s. per foot? 9s×2=18l. Ans.

If the given price of 1 foot be pence only, or shillings and pence, divide the given price, in pence, by 6. The quotient will be the answer in pounds, and the remainder will be so many times 3s. 4d.

7. What cost 40 feet; at 17d. per foot?

6)17

£2 16 8 Ans.

8. At 1s. 9d. per foot, what cost a ton?

6)21

£3 10 Ans.

If the given price be farthings only, or pence and farthings, divide the given price, in farthings, by 6; then

divide *that quotient* by 4, and *this last quotient* will be the answer.

9. At 3qrs. per foot, what cost a ton?

6)3

4)0 10

£0 2 6 Ans.

10. At $13\frac{1}{4}$d. per foot, what cost a ton?

$13\frac{1}{4}$
4

6)53

4)8 16 8

£2 4 2 Ans.

Or, Suppose every shilling in the price to be £2, every penny to be 3s. 4d. and every farthing to be 10d.

11. What cost 40 feet, at 3qrs. per foot?

3qrs.×10=2s. 6d. Ans.

12. What cost 40, at $15\frac{1}{2}$d. per foot?

s.	d.			
1	0×	2=	£2	0 0
3	4×	3=	0	10 0
0	½×	10=	0	1 8

£2 11 8

III.* *To find the value of Goods sold by 5 score to the Hundred.*

1st. If the given price be pounds and shillings, or shillings only,

RULE.

Multiply the given price, in shillings, by 5, and the product will be the answer in pounds.

* *In Federal Money.*—Remove the decimal point two places to the right for the answer.

EXAMPLES.

1. What cost 100 yards, at D.2 50c. per yard?
D 2 50×100=D.250, Ans.
2. What cost 100 yards, at 75c. per yard?
D.0 75×100=D.75, Ans.
3. What cost 100 yards, at 5c. $6\frac{1}{4}$m. per yard?
D.0·05625×100=D.5·625, Ans.
4. What cost 100 yards, at 37c. 5m. per yard?
Ans. 37dols. 50c.
5. What cost 100 yards, at 68c. $7\frac{1}{2}$m. per yard?
Ans. 68dols. 75c.

13. At 19s. per yard, what cost 100 yards?

19s.
5
—
£95 Ans.

14. At £4 13s. per cwt. what cost 100 cwt. or 5 tons?

4 13
20
—
93
5
—
£465 Ans.

2d. If the given price of 1 be pence only, or shillings and pence,

RULE.

Multiply the given price, in pence, by 5; then divide that product by 12. The quotient will be pounds; and the remainder so many times 1s. 8d.

15. If 1 yard cost 9d. what cost 100 yards?

9
5
—
12)45
—
£3 15 Ans.

16. What cost 100 bushels, at 35s. 4d. per bushel?

s. d.
35 4
12
—
424
5
—
12)2120
—
£176 13 4 Ans.

Or,

35s. 4d.
| 4d. | ⅓ | 5
—
175
1 13 4
—
£176 13 4

Here 5 is divided by ⅓.

3d. If the given price of 1 be shillings and pence: Multiply the price by 5, and the product under the place of shillings will be the answer in pounds, and the product under the place of pence, will be so many times 1s. 8d.

17. At 2s. 5d. per bushel, what cost 100 bushels?

s.	d.
2	5
	5
12	1

£12 1 8 Ans.

18. At 25s. 3d. per ton, what cost 100 tons?

s.	d.	
25	3	1s. 8d×3=5s.
	5	
126	3	

£ 126 5 Ans.

4th.* To find the price of 1, at so much per hundred of 5 score.

GENERAL RULE.

Multiply the given price by 12; divide the product by 5, and the quotient will be the answer in pence.

But if the price be pounds only:

RULE.

Divide the given price by 5, and the quotient will be the answer in shillings.

19. If 100yds. cost £65 what cost 1 yard?

5)65

13s. Ans.

20. If 100yds. cost £2 18s. 4d. what is that per yard?

£	s.	d.
2	18	4
		12
5)35	0	0

7d. Ans.

21. If 100 yards cost £11 7s. 9d. what cost 1 yard?

£	s.	d.
11	7	9
		12
5)136	13	0
12)27	6	7

2s. 3¼d. Ans.

In dividing 27 by 12 (in the 21st question) the quotient is 2s. and the remainder 3d. the 6 is $\frac{6}{24}$ of a penny,=one farthing, and the 7 is of no account.

* *In Federal Money.* Remove the decimal point two places to the left for the answer.

EXAMPLES.

1. If 100 yards cost D.250, what cost 1 yard?

D.250÷100=D.2·50 Ans.

2. If 100 yards cost D.75, what cost 1 yard?

D.75÷100=D.0·75, Ans.

TABLE of Aliquot Parts. 100 the Integer.

5 =	$\frac{1}{20}$	25 =	$\frac{1}{4}$	50 =	$\frac{1}{2}$	75 =	$\frac{3}{4}$
10 =	$\frac{1}{10}$	30 =	$\frac{2}{10}$	60 =	$\frac{6}{10}$	80 =	$\frac{8}{10}$
20 =	$\frac{1}{5}$	40 =	$\frac{4}{10}$	70 =	$\frac{7}{10}$	90 =	$\frac{9}{10}$

22.* At 3l. 7s. 6d. per hundred, what will 23 cost?

		£	s.	d.	
20	$\frac{1}{5}$	3	7	6	
		0	13	6	}
2	$\frac{1}{10}$	0	1	4	} Add.
1	$\frac{1}{2}$	0	0	8	}

23 = £0 15 6 Answer.

23. At 2l. 1s. 10d. per 100, what cost 18?

		£	s.	d.	
20	$\frac{1}{5}$	2	1	10	
		0	8	$4\frac{2}{5}$	}
2	$\frac{1}{10}$	0	0	10	} Sub.
18	=	£0	7	$6\frac{2}{5}$	Ans.

24. At 5l. 9s. 6d. per 100, what cost 35?

		£	s.	d.	
		5	9	6	
				3	
10)		16	8	6	
5	$\frac{1}{8}$	1	12	10	}
			5	$5\frac{2}{5}$	} Add
		£1	18	$3\frac{2}{5}$	Ans.

3. If 100 yards cost D 5 62c. 5m. what cost 1 yard?
D.5·625÷100=D.05625=5c. $6\frac{1}{4}$m. Ans.

4. If 100 yards cost D.37 50c. what cost 1 yard? Ans. 37c. 5m.

5. If 100 yards cost D.68 75c. what cost 1 yard? Ans. 68c. $7\frac{1}{2}$ m.

* *To find the value of any number at a given price per* 100 *in Federal Money*—Multiply the price per 100 by the given quantity, and point off two right hand figures, in the product, more than required by multiplication of decimals. Or, point off the two right hand places in the given quantity, and multiply and point, as in multiplication of decimals.

EXAMPLES.

1. What cost 56 yards, at D.87 50c. per 100 yards?

$$\frac{D.87{\cdot}5 \times 56}{100} = D.49 \text{ Ans.}$$ Or, D.87·5×·56=D.49 as before.

2. What cost 45½lb. of beef, at D.5½ per 100?

$$\frac{D.5{\cdot}5 \times 45{\cdot}5}{100} = D.2{\cdot}5025 = D.2\ 50c.\ 2\tfrac{1}{2}m.\ \text{Ans.}$$

Or, D.5·5×·455lb.=D.2·5025, as before.

IV. *To find the value of Goods sold by* 112lb. *the Cwt.*

The price of 1lb. is given to find the value of 1cwt.

RULE.

For a farthing account 2s. 4d. per cwt. For half a penny, 4s. 8d. For 3qrs. 7s. And for every penny 9s. 4d. per cwt.

25. What cost 1cwt. at 3¼d. per cwt.?

At 1d. per lb. s. d.
1cwt. costs 9 4
3

At 3d.	£1	8	0	Add	
At ¼d.	0	4	8		
	£1	12	8	Ans.	

26. At 8¾d. per lb. what cost 1cwt.?

At 1d. per lb. s. d.
1cwt. costs. 9 4
8

At 8d.	£3	14	8	Add	
At ¾	0	7	0		
	£4	1	8	Ans.	

V. *To find the value of Goods sold by* 6 *Score to the Hundred.*

The price of 1 is given, to find the price of 1 hundred.

RULE.

Suppose every penny in the price to be so many pounds, and for every farthing, such a part of a pound, as they are of a penny; then half of that sum will be the answer.

27. At 4¼d. per yard, what cost 120 yards?

£ s.
2)4 10
£2 5 Ans.

28. At 16s. 9¼d. per yard, what cost 120 yards?

s. d.
16 9¼
12
2)201£ 5s.
£100 12 6 Ans.

3. What cost 375 yards, at 375 dollars per 100 yards?
Ans. B.1406 25c.

4. What cost 54 yards, at 16 dollars per 100 yards?
Ans. D.8 64c.

5. What cost 512 yards, at 6 dollars 25 cents per 100 yards?
Ans. D.32.

To find the price of 1, at so much per hundred of 6 score.

RULE.

Multiply the price by 2, then call the pounds so many pence, and the shillings such a part of a penny, as they are of a pound, and you will have the answer.

29. If 120 yards cost 3l. 12s. what cost 1 yard?

£	s.
3	12
	2
7	4

Ans. $7\frac{1}{3}$d,

30. If 120 yards cost 5l. 18s. 6d. what cost 1 yard?

£	s.	d.
5	18	6
		2
11	17	0

Ans. $11\frac{2}{3}$d. + $\frac{2}{3}$qr. Ans.

TABLE of Aliquot Parts. 120 the integer.

6	= $\frac{1}{20}$	24	= $\frac{1}{5}$	36	= $\frac{3}{10}$	72	= $\frac{3}{5}$	96	= $\frac{4}{5}$		
10	= $\frac{1}{12}$	30	= $\frac{1}{4}$	45	= $\frac{3}{8}$	75	= $\frac{5}{8}$	100	= $\frac{5}{6}$		
12	= $\frac{1}{10}$	40	= $\frac{1}{3}$	48	= $\frac{2}{5}$	80	= $\frac{2}{3}$	105	= $\frac{7}{8}$		
15	= $\frac{1}{8}$	60	= $\frac{1}{2}$	50	= $\frac{5}{12}$	84	= $\frac{7}{10}$	108	= $\frac{9}{10}$		
20	= $\frac{1}{6}$			70	= $\frac{7}{12}$	90	= $\frac{3}{4}$	110	= $\frac{11}{12}$		

Also,

31. At 3l. 17s. 6d. per hundred, what cost 14?

		£	s.	d.	
12	$\frac{1}{10}$	3	17	6	
		0	7	9	Add
2	$\frac{1}{6}$	0	1	$3\frac{1}{2}$	Add
14	=	£0	9	$0\frac{1}{2}$	Ans.

32. At 2l. 13s. $6\frac{1}{2}$d. per hundred, what cost 49?

		£	s.	d.
40	$\frac{1}{3}$	2	13	$6\frac{1}{2}$
8	$\frac{1}{5}$	0	17	$10\frac{2}{3}$
1	$\frac{1}{8}$	0	3	$6\frac{1}{3}$
		0	0	5 1
49	=	£1	1	10 1 Ans.

33. At £1 19s. 3d. per hundred, what cost 75?

£	s.	d.
1	19	3
		5
8)9	16	3
£1	4	$6\frac{1}{4}$

VI. *To find the value of Goods sold by the Great Gross.*

NOTE. 12 make 1 dozen, 12 dozen 1 small gross, 12 small gross 1 great gross.

The price of 1 dozen being given, in pence to find the price of a great gross.

RULE.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c.

For proof do the contrary.

N. B. If the price of 1 be given, the price of one small gross is found after the same manner.

34. What cost 1 great gross, at 18d. per dozen?

3

5)54

£10 16

35. At 4s. 3d. per doz. what cost 1 great gross?

4s. 3d.	Or,	Or,
12	s. d.	£ s.
—	4 3	144=7 4
51d.	12	4
3	—	—
—	2 11 0	3d. \| ¼ \| 28 16 } Add
5)153	12	1 16 }
—	—	—
£30 12 Ans.	£30 12 0	£30 12

TABLE of Aliquot Parts. 144 the Integer.

Also,

12 = 1/12	36 = 1/4	32 = 4/9	84 = 7/12	128 = 8/9			
16 = 1/9	48 = 1/3	60 = 5/12	96 = 2/3	132 = 11/12			
18 = 1/8	72 = 1/2	64 = 1/2 1/8	108 = 3/4				
24 = 1/6		80 = 5/9	120 = 5/6				

36. At 2l. 12s. 9d. per great gross, what cost 45 dozen?

		£	s.	d.
36	¼	2	12	9
		0	13	2¼ } Add.
9	¼	0	3	3½ }
45 =		£0	16	5¾ Ans.

37. What cost 117 dozen, at 0l. 13s. 7d. per great gross?

		£	s.	d.
		9	13	7
			8	
Doz.				
108	⅔	29	0	9
9	1/12	7	5	2¼ } Add
			12	1 }
117	=	£7	17	3¼ Ans.

38. At 3l. 16s. 8d. per great gross, what cost 7 great gross and 96 dozen?

	£	s.	d.
	3	16	8
			2
	3)7	13	4
Doz.			
96 =	2	11	1¼
Top line ×7 =	26	16	8
	£29	7	9¼

VII.* *To find the value of Goods sold by the Thousand.*

The price of 1 is given to find the price of 1000.

RULE.

Multiply the given price, in pence, by 50, then divide the product by 12, and the quotient will be the answer in pounds, &c.

39. At 6d. each, what cost 1000?

6
50
12)300
£25 Ans.

Or, as 1000s. are 50l. take parts for the pence out of 50.

6d.	½	50
		Ans. 25

40. What cost 1000, at 2¼d. each?

2d.	⅙	50
¼	⅛	8 6 8 } +
		1 2 10 }
		£9 7 6 Ans.

* *In Federal Money*—Remove the decimal point three places to the right or left, as the case requires, for the answer.

EXAMPLES.

1. What cost 1000 yards, at 5 cents per yard?

·05 × 1000 = 050 = D.50, Ans.

VIII.* *To find the price of one at so much per thousand.*

RULE.

Multiply the price by 12; divide the product by 50; then take the pounds for so many pence, and the shillings for such a part of a penny as they are of a pound, which will be the answer.

41. At 5l. 4s. 2d. per 1000, what cost 1 ?

£	s.	d.
5	4	2
		12

50 { 5)62 10 0 / 10)12 10

£1 5

Ans. 1¼d.

42. At 354l. 3s. 4d. per 1000, what cost 1 ?

£.	s.	d.
354	3	4
		12

50 { 10)4250 0 0 / 5)425

85

Ans. 7s. 1d.

Or,

		£.	s.	d.
100	$\frac{1}{10}$	354	3	4
10	$\frac{1}{10}$	35	8	4
1	$\frac{1}{10}$	3	10	10

Ans. 0 7 4

TABLE of Aliquot Parts. 1000 the Integer.

50 = $\frac{1}{20}$		Also,			
100 = $\frac{1}{10}$	300 = $\frac{3}{10}$		700 = $\frac{7}{10}$		
125 = $\frac{1}{8}$	375 = $\frac{3}{8}$		750 = $\frac{3}{4}$		
200 = $\frac{1}{5}$	400 = $\frac{2}{5}$		800 = $\frac{4}{5}$		
250 = $\frac{1}{4}$	600 = $\frac{3}{5}$		875 = $\frac{7}{8}$		
500 = $\frac{1}{2}$	625 = $\frac{5}{8}$		900 = $\frac{9}{10}$		

2. What cost 1000 yards, at 12 cents 5 mills per yard?

Ans. D.125.

3. If 1000 yards cost D.37 50c. what cost 1 yard?

D.37·5 ÷ 1000 = D.0375 = 3c. 7$\frac{1}{2}$m. or 3¾c. Ans.

4. If 1000 yards cost D.1625, what cost 1 yard?

Ans. D.1 62c. 5m.

* See the preceding Note.

43.* At £1 17s. 9d. per 1000, what cost 115?

		£	s.	d.	
100	$\frac{1}{10}$	1	17	9	
10	$\frac{1}{10}$	0	3	$9\frac{1}{4}$	Add.
		0	0	$4\frac{1}{2}$	
5	$\frac{1}{2}$	0	0	$2\frac{1}{4}$	
115	=	£0	4	4	Ans.

44. At £2 1s. 8d. per 1000, what cost 875?

£ s. d.
2 1 8
7

8)14 11 8

£1 16 $5\frac{1}{2}$ Ans.

45. What cost 33, at 24s. 8d. per 1000?

		£	s.	d.	
50	$\frac{1}{20}$	1	4	8	
25	$\frac{1}{2}$	0	1	$2\frac{3}{4}$	
		0	0	$7\frac{1}{4}$	Add.
5	$\frac{1}{5}$	0	0	$1\frac{1}{4}$	
30	=	0	0	$8\frac{1}{2}$	Add.
3	$\frac{1}{10}$	0	0	$0\frac{2}{4}$	
33	=	0	0	$9\frac{1}{4}$	

* *To find the value of any number, at a given price per thousand, in Federal Money*—Multiply the price per 1000 by the given quantity, and point off three right hand figures in the product more than required by multiplication of decimals. Or, point off the three right hand places in the given quantity; and multiply and point as in multiplication of decimals.

EXAMPLES.

1. What cost 875, at D.13 per 1000.
 875 × 13 = 11375 ÷ 1000 = 11·375 = D.11 37c. 5m. Ans.
2. What cost 39175 feet of boards, at D.16 per 1000?
 Ans. D 626 80c.
3. What cost 325 nails, at D.1 50 cents per 1000?
 Ans. 48c. $7\frac{1}{2}$m. or, $48\frac{3}{4}$c.

CASES IN FEDERAL MONEY.

CASE I.

When the price of 1 *is an aliquot part of a dollar*:—Divide the quantity by the denominator of the fraction, which the price is of a dollar, and the quotient will be the answer in dollars, &c.

EXAMPLES.

1. What cost 227 yards, at 50 cents per yard?

c. D. D.
50 | ½ | 227=price at D.1 per yard.

D.113 50c. Answer.

2. What cost 265 yards, at 12c. 5m. per yard?

Ans. D.33 12c. 5m.

3. What cost 269½ yards, at 16⅔c. per yard?

16⅔c. | ⅙ | 269·5 price at D.1.

D.44 91c. 6⅔m. Ans.

4. What cost 1050 yards, at 6¼c. per yard?

Ans. D.65 62c. 5m.

CASE II.

When the price of 1 *is two or more aliquot parts of a dollar added together*: Divide the given number first for one aliquot part, then for another, &c. the quotients added together, will be the answer.

EXAMPLES.

1. What cost 298 yards, at 75 cents per yard?

c.	D.	D.		
50	½	298	price at D.1	
25	¼	149	ditto at	·50
		74 50	ditto at	·25
		Ans. 223 50	ditto at	·75

2. What cost 927 yards, at 53⅓c. per yard?

Ans. D.494 40c.

3. What cost 618 yards, at 87½c. per yard?

Ans. D.540 75c.

4. What cost 328 yards, at 57c. 5m. per yard?

Ans. D.188 60c.

CASE III.

When the price of 1 *is the difference between two aliquot parts of a dollar:* Find the price at the greater aliquot part, and then at the less, and their difference will be the answer.

EXAMPLES.

1. What cost 328 yards, at 13⅓ cents per yard?

c.	D.	D.			
33⅓	⅓	328	price at D.1		
20	⅓	109 33⅓	ditto at	·33⅓c.	
		65 60	ditto at	·20	
		Ans. 43 73⅓	ditto at	13⅓c.	

2. What cost 817 yards, at 30c. per yard?

Ans. D.245 10c.

3. What cost 296 yards, at 15 cents per yard?

Ans. D.44 40c.

CASE IV.

When the price of 1 *is any sum less than a dollar:* Divide the given price into aliquot parts, either of a dollar, or of each other: find the price at each, and add them together for the answer.

EXAMPLES.

1. What cost 279 yards, at 31 cents per yard?

25c.	¼D.	279·	price at D.1	
5	⅕ of 25c.	69·75	ditto at	25c.
1	⅕ of 5	13·95	ditto at	5
		2·79	ditto at	1
		Ans. D.86·49	ditto at	31c.

2. What cost 953 yards, at 57c. per yard?

Ans. D.543 21c.

3. What cost 839 yards, at 36c. per yard?

Ans. D.302 4c.

CASE V.

When the price of 1 *is any sum between* 1 *and* 2 *dollars:* The quantity itself in dollars, is the price at D.1; then

finding, by the preceding rules, the price at the parts of D.1, the sum of the whole is the answer.

EXAMPLES.

1. What cost 386 yards, at D.1·65c. per yard?

50c.	½D.	386	price at D.1	
10	⅕ of 50c.	193	ditto at	50c.
5	½ of 10	38 60	ditto at	10
		19 30	ditto at	5
		Ans. D.636 90	ditto at	D.1 65c.

2. What cost 849 yards, at D.1·72 per yard?
Ans. D.1460·28

3. What cost 294 yards, at D.1·18 per yard?
Ans. D.346·92.

CASE VI.

When the price of 1 *is any number of dollars and parts of a dollar:* Multiply the quantity by the number of dollars; and, finding, by the preceding rules, the price at the parts of D.1, the sum of the whole is the answer.

EXAMPLES.

1. What cost 395 yards, at D.3 24c. per yard?

c.		D. c.		
20	⅕D.	395	price at D.1	
		3		
4	⅕ of 20c.	1185	ditto at	3
		79	ditto at	20c.
		15 80	ditto at	4
		Ans. D.1279·80	ditto at	D.3 24c.

2. What cost 269 yards at D.2 60c. per yard?
Ans. D.699 40c.

3. What cost 694 yards, at D.12·10 per yard?
Ans. D.8397 40c.

4. What cost 318 yards, at D.4·125 per yard?
Ans. D.1311 75c.

5. What cost 175 yards, at D.4·44 per yard?
Ans. D.777.

CASE VII.

When the price of 1 *contains the same aliquot part of a dollar any number of times exactly; or, in other words, when the price of* 1 *has an aliquot part, which is also an aliquot part of a dollar:* First, find the value of the given quantity at the aliquot part; then multiply this by the number of times, which the aliquot part is contained in the given sum, for the answer.

Or, Since the price in this case is always such a number, as, being divided by the aliquot part, will make the numerator of a fraction, of which the denominator is the denominator of that fraction, which the aliquot part is of a dollar: Multiply the quantity by the numerator, and divide the product by the denominator, (or, when convenient, divide the quantity by the denominator, and multiply the quotient by the numerator,) for the answer.

EXAMPLES.

1. What cost 384 yards, at $87\frac{1}{2}$ cents per yard?

$12\frac{1}{2}$c.$=\frac{1}{8}$ of $\cdot 875 = \left|\frac{1}{8}\text{Dr}\right| 384\cdot$=price at D.1

$48\cdot$=ditto at $\cdot 12\frac{1}{2}$

$\times 7$ $\times 7$

Ans. D.336$\cdot$=ditto at $\cdot 87\frac{1}{2}$

Or thus,

$$\cdot 875 = \text{D.}\frac{7}{8}, \text{ and } 384\times\frac{7}{8} = \frac{384\times 7}{8} \left(=\frac{2688}{8}\times 7\right) = \text{D.}336 \text{ answer as before.}$$

2. What cost 842 yards, at $66\frac{2}{3}$c. per yard?

Ans. D.561 $33\frac{1}{3}$c.

3. What cost 912 yards, at 55 cents per yard?

Ans. D.501 60c.

MISCELLANEOUS QUESTIONS.

1. What cost 300 yards, at 27 cents per yard?

Ans. D.81.

2. What cost 917 yards, at D.1·125 per yard?

Ans. D.1031 62c. 5m.

3. What cost $35\frac{1}{2}$ yards, at 35 cents per yard?

Ans. D.12. 32c.

4. What cost 862½ feet of boards, at D.12 per M.?
Ans. D.10 34c. 6m.
5. What cost 32159ft. boards, at D.13·75 per M.?
Ans. D.442 18c. 6¼m.

BILL OF PARCELS.

Newburyport, Jan. 1st. 1809.

Mr. Timothy Huckster

Bought of *Samuel Merchant*,

25½lb. Bohea Tea, at 3s. 6d. per lb.
48lb. Cheese, at 9d. per lb.
15 Pair Worsted Hose, at 5s. 8d. per pair,
4½ Dozen Women's Gloves, at 36s. 6d. per dozen,
19 Dozen Knives and Forks, at 5s. 9d. per dozen,
9 Grindstones, at 15s. 9d. per Stone,
¼ Cwt. Brown Sugar, at 51s. per cwt.
31 lb. Loaf Sugar, at 1s. 0½d. per lb.

£34 3 3½

Received payment in full,
Samuel Merchant.

TARE AND TRET.

TARE and Tret are practical Rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of 4lb. in every 104lb. for waste, dust. &c.

Cloff is an allowanceof 2lb. upon every 3cwt.

Gross weight is the weight of any sort of goods, together with the box, barrel, or bag, &c. which contains them.

Suttle is when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made

CASE I.*

When the tare is at so much per box, barrel, or bag, &c.: **Multiply the number of boxes, barrels, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight required.**

EXAMPLES.

1. In 6 hogsheads of sugar, each weighing 9cwt. 2qrs. 10lb. gross, tare 25lb. per hogshead; how much neat?

	Cwt.	qr.	lb.	Cwt.	qr.	lb.	
25×6=1	1	1	10	9	2	10	gross weight of 1 hhd.
						6	
				57	2	4	gross.
				1	1	10	tare.
				Ans. 56	0	22	neat

2. In 5 bags of cotton, marked with the gross weight as follows, tare 23lb. per bag, what neat weight?

	Cwt.	qr.	lb.
A =	7	1	19
B =	3	3	27
C =	5	1	12
D =	6	0	15
E =	8	1	0

Ans. 30cwt. 0qr. 14lb. neat.

3. What is the neat weight of 15 hogsheads of tobacco, each weighing 7cwt. 1qr. 13lb. tare 100lb. per hhd.?

Ans. 97cwt. 0qr. 11lb.

CASE II.

When the tare is at so much per cwt. Divide the gross weight by the aliquot parts of a cwt. subtract the quotient from the gross, and the remainder will be the neat weight.

EXAMPLES.

1. In 129cwt. 3qrs. 16lb. gross, tare 14lb. per cwt. what neat weight?

* This, as well as every other case in this rule, is only an application of the rules of Proportion and Practice.

		Cwt.	qr.	lb.	
14lb.	$\frac{1}{7}$	129	3	16	gross.
		16	0	20$\frac{1}{2}$	tare.
		Ans. 113	2	17$\frac{1}{2}$	neat.

2. In 97cwt. 1qr. 7lb. gross, tare 20lb. per cwt. what neat weight?

		Cwt.	qr.	lb.	
16	$\frac{1}{4}$	97	1	7	gross.
4	$\frac{1}{4}$	13	3	17	Add.
		3	1	25	
		Subtract 17	1	14	tare.
		Ans. 79	3	21	neat.

3. What is the neat weight of nine barrels of pot-ash, each weighing 305lb. gross, tare 12lb. per cwt.?

Ans. 2450lb. 14oz. 4$\frac{4}{7}$dr.

4. What is the value of the neat weight of 7 hogsheads of tobacco, at 5l. 7s. 6d. per cwt. each weighing 8cwt. 3qrs. 10lb. gross, tare 21lb. per cwt.?

Ans. 270l. 4s. 4$\frac{1}{2}$d. reckoning the odd ounces.

CASE III.

When tret is allowed with tare: Divide the suttle weight by 26, and the quotient will be the tret, which subtract from the suttle, and the remainder will be the neat?

Examples.

1. In 247cwt. 2qrs. 15lb. gross, tare 28lb. per cwt. and tret 4lb. for every 104lb. what neat weight?

		Cwt.	qr.	lb.	oz.	
28	$\frac{1}{4}$	247	2	15		gross.
		61	3	17	12	tare subtract.
4	$\frac{1}{26}$	185	2	25	4	suttle.
		7	0	16	0	tret, subtract.
		Ans. 178	2	9	4	neat.

2. What is the neat weight of 4 hogsheads of tobacco, weighing as follow:

The 1st	5cwt.	1qr.	12lb. gross,	tare 65lb.
2d	3	0	19	75
3d	6	3	00	49
4th	4	2	9	35

and allowing tret to each as usual?

Ans. 17cwt. 0qr. 19lb. +

CASE IV.

When tare, tret, and cloff are allowed: Deduct the tare and tret as before, and divide the suttle by 108, and the quotient will be the cloff, which subtract from the suttle, and the remainder will be the neat.

EXAMPLES.

1. What is the neat weight of 1hhd. of tobacco, weighing 16cwt. 2qrs. 20lb. gross, tare 14lb. per cwt. tret 4lb. per 104, and cloff 2lb. per 3cwt.?

14lb. is $\frac{1}{8}$)16	2	20	0	gross.	
	2	0	9	8	tare subtract.
4lb. is $\frac{1}{28}$)14	2	10	8		
	2	6	13	tret, subtract.	
2lb. is $\frac{1}{168}$)14	0	3	11	suttle.	
	0	9	5	cloff, subtract.	
Ans.	13	3	22	6	neat.

2. If 9 hogsheads of tobacco contain 85cwt. oqr. 21lb. tare 30lb. per hogshead, tret and cloff as usual, what will the neat weight come to at 6½d. per lb. after deducting for duties and other charges £51 11s. 8d.?

Ans. £187 18s. 5d.

INVOLUTION,

OR TO RAISE POWERS.

A POWER is the product arising from multiplying any *given* number into itself continually a certain number of *times, thus:*

$3\times3=9$ is the 2d power, or square of 3 $=3^2$.
$3\times3\times3=27$ is the 3d power or cube of 3 $=3^3$.
$3\times3\times3\times3=81$ is the 4th power, or the biquadrate of 3, &c. $=3^4$.

The number denoting the power is called the *index*, or the *exponent* of that power. Thus, the 4th power of 3 is 81, or 3^4; the second power of 5 is 25, or 5^2, &c.

$2\times2=4$, the square of 2; $4\times4=16=4$th power of 2; $16\times16=256=8$th power of 2, &c.

RULE.

Multiply the given number, root, or first power, continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

NOTE. Whence, because fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved by raising each of their terms to the power required, and if a mixed number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

EXAMPLES.

1. What is the 5th power of 9?

9
—
81=2d power.
9
—
729=3d power.
9
—
6561=4th power.
9
—
59049=5th power, or answ.=9^5.

2. What is the 5th power of $\frac{3}{5}$? Ans. $\frac{243}{3125}$.

3. What is the fourth power of ·045?
Ans. ·000004100625.

Here we see, that in raising a fraction to a higher power, we decrease its value.

A TABLE OF POWERS.

Roots	1	1	2	3	4	5	6	7	8	9
Squares	2	1	4	9	16	25	36	49	64	81
Cubes	3	1	8	27	64	125	216	343	512	729
Biquadrates	4	1	16	81	256	625	1296	2401	4096	6561
Sursolids	5	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes	6	1	64	729	4096	15625	46656	117649	262144	531441
Second Sursolids	7	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates squar'd	8	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes cubed	9	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Sursolids squared	10	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Sursolids	11	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Squa. Cube squared	12	1	4096	531441	16777216	244140625	2176782336	13841287001	68719476736	282429536481

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

THE Root is a number, whose continued multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d, 3d, 4th, &c. root, accordingly as it is, when raised to the 2d, 3d, &c. power, equal to that power. Thus, 4 is the square root of 16, because $4\times4=16$, and 3 is the cube root of 27, because $3\times3\times3=27$, and so on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which pre cise roots can never be determined. But by the help of decimals, we can approximate towards the root, to any assigned degree of exactness.

The roots which aproximate, are called *surd roots;* and those which are perfectly accurate, are called *rational roots.*

Roots are sometimes denoted by writing the character $\surd$ before the power, with the index of the power over it; thus, the 3d root of 36 is expressed $\surd^{3}$ 36, and the 2d root of 36 is $\surd$ 36, the index 2 being omitted when the square root is designed.

If the power be expressed by several numbers, with the sign + or — between them, a line is drawn from the top of the sign over all the parts of it. Thus the 3d root of 47+22 is $\sqrt[3]{47+22}$, and the 2d root of 59—17 is $\surd\overline{59-17}$, &c.

Sometimes roots are designed like powers, with fractional indices. Thus the squareroot of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and the 4th root of 37—20 is $\overline{37-20}^{\frac{1}{4}}$, &c.

EXTRACTION OF THE SQUARE ROOT.

RULE.

1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor: Multiply the divisor, with the figure last annexed, by the figure last placed in the root, and subtract the product from the dividend: To the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new divisor, (or bring down your last divisor for a new one, doubling the right hand figure of it) and from these find the next figure in the root as last directed; and continue the operation in the same manner till you have brought down all the periods.

NOTE 1. If when the given power is pointed off as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

NOTE 2. If there be decimals in the given number, it must be pointed both ways from the place of units: If when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many cyphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing cyphers.

EXAMPLES.

1. Required the square root of 30138696025.

```
                30138696025(173605 the root.
                1
                ---
1st Divisor=27)201
                189
                ---
2d Divisor=343)1238
                1029
                ----
3d Divisor=3466)20969
                20796
                -----
4th Divisor=347205)1736025
                   1736025
                   -------
```

2. Required the square root of 575·5.

```
          575·50(23·98 +, root.
          4
          ---
    43)175
       129
       ---
   469)4650
       4221
       ----
  4788)42900
       38304
       -----
        4596 Remainder.
```

3. What is the square root of 10342656 ?

Ans. 3216.

4. What is the square root of 964·5192360241 ?

Ans. 31·05671.

5. What is the square root of ·0000316969 ?

Ans. ·00563.

NOTE. When more than half the root is found, the remaining figures of it may be found by Division, making use of the last divisor, and bringing down so many of the next figures of the resolvend, as there are periods to come down when you began the division.

RULES *For the Square Root of Vulgar Fractions and Mixed Numbers.*

After reducing the fraction to its lowest terms, for this and all other roots; then,

1. Extract the *root* of the *numerator* for a *new numerator*, and the *root* of the *denominator* for a *new denominator*, which is the best method, provided the denominator be a complete power. But if it be not,

2. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required: Or,

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed numbers may either be reduced to improper fractions and extracted by the first or second rule; or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

EXAMPLES.

1. What is the square root of $\frac{144}{1681}$?

By Rule 1.

$\frac{144}{1681} = \frac{16}{1681}$

16(4 root of the numerator.
16

1681(41 root of the denominator.
16

81)81 Therefore, $\frac{4}{41}$ = the root of the given fraction.
81

By Rule 2.

$16 \times 1681 = 26896$, and $\sqrt{26896} = 164$. Then

$\frac{144}{1681} = \frac{16}{164} = \frac{4}{41} = .09756+$

By Rule 3.

1681)16(·0095181439+ And $\sqrt{\cdot 0095181439} = \cdot 09756+$.

2. What is the square root of $\frac{4713}{1331}$? Ans. $\frac{7}{11}$.

3. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.

NOTE. In extracting the square or cube root of any surd number there is always a remainder or fraction left when the root is found: To find the value of which, the common method is to annex pairs of cyphers to the resolvend for the square, and ternaries of cyphers to that of the cube, which makes it tedious to discover the value of the remainder, especially in the cube. Now this trouble may be saved by the following method.

In the square the quotient is always doubled for a new divisor: Therefore when the work is completed, the root doubled is the true divisor, or denominator* to its own fraction; as, if the root be 12, the denominator will be 24; to be placed under the remainder; which vulgar fraction, or its equivalent decimal, must be annexed to the quotient, or root, to complete it.

If to the remainder, either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

APPLICATION AND USE OF THE SQUARE ROOT.

PROBLEM I. *To find a mean proportioned between two numbers.*

RULE. Multiply the given numbers together, and extract the square root of the product; which root will be the mean proportional sought.

EXAMPLE.

What is the mean proportional between 24 and 96?

$\sqrt{96 \times 24} = 48$. Answer.

* These denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use.

PROB. II. *To find the side of a* square *equal in* area *to any given* superficies *whatever.*

RULE. Find the *area*, and the square root is the side of the square sought.

EXAMPLES.

1. If the area of a circle be 184·125, what is the side of a square equal in area thereto?

$\sqrt{184{\cdot}125}=13{\cdot}569+$ Answer.

2. If the area of a triangle be 160, what is the side of a square equal in area thereto?

$\sqrt{160}=12{\cdot}649+$ Answer.

PROB. III. A certain general has an army of 5625 men: Pray, how many must he place in rank and file, to form them into a square? $\sqrt{5625}=75$ Ans.*

PROB. IV. Let 10952 men be so formed, that the number in rank may be double the file.

$\sqrt{\frac{10952}{2}}=74$ in file, and $74\times2=148$ in rank.

PROB. V. If it be required to place 2016 men so that there may be 56 in rank and 36 in file, and to stand 4 feet distance in rank, and as much in file; how much ground do they stand on?

To answer this, or any of the kind, use the following proportion:—As unity : to the distance :: so is the number in rank less by one : to a fourth number;—next do the same by the file, and multiply the two numbers together, found by the above proportion, and the product will be the answer.†

As 1 : 4 :: 56—1 : 220. And, as 1 : 4 :: 36—1 : 140. Then $220\times140=30800$ square feet, the answer.

PROB. VI. Suppose I would set out an orchard of 600 trees, so that the length shall be to the breadth, as 3 to 2, and the distance of each tree, one from the other, 7 yards;

* If you would have the number of men double, triple, or quadruple, &c. as many in rank as in file, extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the given number of men, and that will be the number of men in file, which double, trible, quadruple, &c and the product will be the number in rank.

† The above rule will be found useful in planting trees, having the distance of ground between each given.

how many trees must it be in length, and how many in breadth; and how many square yards of ground do they stand on?

To resolve any question of this nature, say, as the ratio in length : is to the ratio in breadth :: so is the number of trees : to a fourth number; whose square root is the number in breadth; And as the ratio in breadth : is to the ratio in length :: so is the number of trees to a fourth, whose root is the number in length.

As 3 : 2 :: 600 : 400. And $\sqrt{400}=20$=number in breadth.

As 2 : 3 :: 600 : 900. And $\sqrt{900}=30$=number in length.

As 1 : 7 :: 30—1 : 203. And as 1 : 7 :: 20—1 : 133. And $203\times133=26999$ square yards. Ans.

Prob. VII. Admit a leaden pipe $\frac{3}{4}$ inch diameter will fill a cistern in 3 hours; I demand the diameter of another pipe, which will fill the same cistern in 1 hour.

Rule. As the given time is to the square of the given diameter, so is the required time to the square of the required diameter.

$\frac{3}{4}=.75$: and $.75\times.75=.5625$. Then, as 3h. : $.5625$:: 1h. : 1.6875 inversely, and $\sqrt{1.6875}=.13$ inch nearly, Answer.

Prob. VIII. If a pipe, whose diameter is 1·5 inch, fill a cistern in 5 hours, in what time will a pipe, whose diameter is 3·5 inches, fill the same?

$1.5\times1.5=2.25$; and $3.5\times3.5=12.25$. Then, as, 2·25 : 5 :: 12·25 : ·918+ hour inversely,=55 minutes 5 seconds, Answer.

Prob. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water; in what time will 3 pipes, each 4 inches bore, be in discharging double the quantity?

$6\times6=36$. $4\times4=16$, and $16\times3=48$. Then as 36 : 4h. :: 48 : 3h. inversely, and as 1w. : 3h. :: 2w. : 6h. Answer.

Prob. X. Given the diameter of a circle to make another circle, which shall be 2, 3, 4, &c. times greater or less than the given circle.

RULE.—Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the root of the product will be the required diameter;—But if the required circle be less; divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle, whose diameter is 4 inches: I demand the diameter of a circle 3 times as large.

$4\times4=16$; and $16\times3=48$; and $\sqrt{48}=6.928+$ inches, Answer.

PROB. XI. To find the diameter of a circle equal in area to an ellipsis (or oval) whose transverse and conjugate diameters are given.*

RULE.—Multiply the two diameters of the ellipsis together; and the Square Root of that product will be the diameter of a circle equal to the ellipsis.

Let the transverse diameter of an elliplis be 48, and the conjugate 36: What is the diameter of an equal circle?

$48\times36=1728$, and $\sqrt{1728}=41.569+$ the Answer.

NOTE. The square of the hypothenuse, or the longest side of a right angled triangle, (by 47th B. 1 Euc.) is equal to the sum of the squares of the other two sides; and consequently the difference of the squares of the hypothenuse and either of the other sides is the square of the remaining side.

PROB. XII. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad. The height of the wall is required?

$36\times36=1296$; and $24\times24=576$. Then, $1296-576=720$, and $\sqrt{720}=26.83+$ yards, the Answer.

PROB. XIII. The height of a tree, growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water is 256 feet. What is the breadth of the stream, provided the land on each side of the water be level?

* The transverse and conjugate are the longest and shortest diameters of an ellipsis; they pass through the centre, and cross each other at right angles.

256×256=65536; and 75×75=5625: Then 65536—5625=59911, and $\sqrt{59911}$=244·76+ and 244·76—$\frac{1}{2}$= 222·76 feet, Answer.

PROB. XIV. Suppose a ladder 60 feet long be so planted as to reach a window 37 feet from the ground, on one side of the street, and without moving it at the foot, will reach a window 29 feet high on the other side; I demand the breadth of the street.

60×60=3600. 37×37=1369. 23×23=529. Then, 3600—1369=2231, and $\sqrt{2231}$=47·23+, and 3600—529=3071, and $\sqrt{3071}$=55·41+, then 47·23+55·41= 102·64 feet. Ans.

PROB. XV. Two ships sail from the same port: one goes due north 45 leagues, and the other due west 76 leagues: How far are they asunder?

45×45=2025, 76×76=5776. Then, 5776+2025= 7801 and $\sqrt{7801}$=88·32 leagues, the Answer.

EXTRACTION OF THE CUBE ROOT.

A Cube is any number multiplied by its square. To extract the cube root, is to find a number which being multiplied into its square, shall produce the given number.

RULE.

1. Separate the given number into periods of three figures each, by putting a point over the unit figure and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube thus found, from the said period, and to the remainder bring down the next period, and call this the dividend.

4. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

5. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. Multiply the triple square by the last quotient figure, and write the product under this dividend: multiply the square of the last quotient figure by the triple quotient, and

place this product under the last; under all, set the cube of the last quotient figure and call their sum the subtrahend.

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.

NOTE. The same rule must be observed for continuing the operation and pointing for decimals, as in the square root.

EXAMPLES.

1. Required the cube root of 436036824287.

436036824287(7583 the root.
343

1st Divisor=14910)93036=1st Dividend.

73500
5250
125

78875=1st Subtrahend.

2d Divisor=1689750)14161824=2d Dividend.

13500000
144000
512

136445·12=2d Subtrahend.

3d Divisor=172391940)517312287=3d Dividend.

517107600
204660
27

517312287=3d Subtrahend.

.

The Method of Operation.

7×7×300	=	14700	= 1st. Triple square.
7×30	=	210	= 1st Triple quotient.
		14910	= 1st Divisor.
14700×5	=	73500	
5×5×210	=	5250	
5×5×5	=	125	
		78875	= 1st Subtrahend.
75×75×300	=	1687500	= 2d Triple square.
75×30	=	2250	= 2d Triple quotient.
		1689750	= 2d Divisor.
1687500×8	=	13500000	
2250×8×8	=	144000	
8×8×8	=	512	
		13644512	= 2d Subtrahend.
758×758×300	=	172369200	= 3d Triple square.
758×30	=	22740	= 3d Triple quotient.
		172391940	= 3d Divisor.
172369200×3	=	517107600	
22740×3×3	=	204660	
3×3×3	=	27	
		517312287	= 3d Subtrahend.

2. What is the cube root of 34965783 ? Ans. 327.
3. What is the cube root of 84·604519 ? Ans. 4·39.
4. What is the cube root of ·008649 ? Ans. ·2052+.
5. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finished.

In the extraction of the cube root, the quotient is said to be squared and tripled for a new divisor: But it is not really so, till the triple number of the quotient be added to it; therefore, when the operation is finished, it is but squaring the quotient, or root, then multiplying it by 3, and to that number adding the triple number of the root, when it will become the divisor, or true denominator to its own fraction, which fraction must be annexed to the quotient to complete the root.

Suppose the root to be 12, when squared it will be 144, and multiplied by 3 it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.

BY APPROXIMATION.

1. Assume a root, whose cube will probably be near the given number; then multiply the square of the assumed root by 3, and divide the resolvend by this product; to this quotient add $\frac{2}{3}$ of the assumed root, and the sum will be the true root, or an approximation to it.

2. For each succeeding operation let the last approximated root be the assumed root, and proceeding in this manner, the root may be extracted to any assigned exactness.

1. What is the cube root of 7?

Let the assumed root be 2. Then $\overline{2\times2\times3}=12$ the divisor.

12)7·0(·583; to this add $\frac{2}{3}$ of 2=1·333, &c. that is, ·583+1·333=1·916 approximated root.

Now assume 1·916 for the root. Then, by the second process, the root is $\frac{7}{\overline{3\times1\cdot916}^2}+\frac{2}{3}\times1\cdot196=1\cdot9126$, &c.

2. What is the cube root of 9? Let 2 be the assumed root as before. Then, $\frac{9}{12}+\frac{2}{3}\times2=2\cdot08$ the approximated root.

Now assume 2·08. Then $\frac{9}{\overline{3\times2\cdot08}^2}+\frac{2}{3}\times2\cdot08=2\cdot08008$, &c.

3. What is the cube root of 282? Let 6 be the assumed root. Then $6\times6\times3=108)282(2\cdot611$, &c. and $2\cdot611+\frac{1}{3}$ of $6=6\cdot611$ approximated root. Now assume 6·611, and it will be $6\cdot611\times6\cdot611\times3=131.116)282(2\cdot1507$, &c. and $2\cdot1507+\frac{1}{3}$ of $6\cdot611=6\cdot558$ a farther approximated root.

4. What is the cube root of 1728? Here the assumed root is 10. Then, $10\times10\times3=300)1728(5\cdot76$, and $5\cdot76+\frac{1}{3}$ of $10=12\cdot426$. Now assume 12·426, then $12\cdot426\times12\cdot426\times3=426\cdot2$ 6428)1728(3·732, and $3\cdot732+\frac{1}{3}$ of $12\cdot426=12\cdot014$ a farther approximated root, and so on.

APPLICATION AND USE OF THE CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

RULE.

1. Divide the *greater* by the *less*, and extract the cube root of the quotient.

2. Multiply the root, so found, by the least of the given numbers, and the product will be the *least*.

3. Multiply this product by the same root, and it will give the greatest.

EXAMPLES.

1. What are the two mean proportionals between 6 and 750?

$750\div6=125$, and $\sqrt[3]{125}=5$. Then $5\times6=30$=least, and $30\times5=150$=greatest. Ans. 30 and 150.

Proof. As 6 : 30 :: 150 : 750.

2. What are the two mean proportionals between 56 and 12096? Ans. 336 and 2016.

NOTE. The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

3. If a bullet 6 inches diameter weigh 32lb; what will a bullet of the same metal weigh whose diameter is 3 inches?

$6\times6\times6=216$. $3\times3\times3=27$.

As 216 : 32lb. :: 27 : 4lb. Ans.

4. If a globe of silver of 3 inches diameter be worth £45, what is the value of another globe, of a foot diameter?

$3\times3\times3=27$. $12\times12\times12=1728$.

As 27 : 45 :: 1728 : £2880. Ans.

2. The side of a cube being given, to find the side of that cube which shall be double, triple, &c. in quantity to the given cube.

RULE. Cube your given side, and multiply it by the given proportion between the given and required cube, and the cube root of the product will be the side sought.

5. If a cube of silver, whose side is 4 inches, be worth £50, I demand the side of a cube of the like silver, whose value shall be 4 times as much?

$4\times4\times4=64$, and $64\times4=256$. $\sqrt[3]{256}=6.349+$ inches. Ans.

6. There is a cubical vessel, whose side is 2 feet; I demand the side of a vessel, which shall contain three times as much?

$2\times2\times2=8$, and $8\times3=24$. $\sqrt[3]{24}=2.884=2$ft. $10\frac{3}{8}$in. Ans.

7.* The diameter of a bushel measure being $18\frac{1}{2}$ inches, and the height 8 inches, I demand the side of a cubick box, which shall contain that quantity?

Ans. 12.907+inches.

8. Suppose a ship of 500 tons has 89 feet keel, 36 feet beam, and 16 feet deep in the hold: what are the dimensions of a ship of 200 tons, of the same mould and shape?

$89\times89\times89=704969$=cubed keel.

As 500 : 200 :: 704969 : 281987·6 cube of the required keel.

$\sqrt[3]{281987.6}=65.57$ feet the required keel.

As 89 : 65·57 :: 36 : 26·522=$26\frac{1}{2}$ feet beam, nearly.

As 89 : 65·57 :: 16 : 11·7 feet, depth of the hold, nearly.

3. From the proof of any cable to find the strength of any other.

RULE. The strength of cables, and consequently the weight of their anchors, are as the cubes of their peripheries.

* Multiply the square of the diameter by ·7854, and the product by the height; the cube root of the last product is the answer.

9. If a cable 12 inches about require an anchor of 18cwt. of what weight must an anchor be for a 15 inch cable?

Cwt. Cwt.

As $12\times12\times12 : 18 :: 15\times15\times15 : 35{\cdot}15625$, Ans.

10. If a 15 inch cable require an anchor 35·15625cwt. what must the circumference of a cable be, for an anchor of 18cwt.?

As $35{\cdot}15625 : 15\times15\times15 :: 18 : 1728$, and $\sqrt[3]{1728}=12$, Answer.

EXTRACTION OF THE BIQUADRATE ROOT.

RULE.

Extract the square root of the resolvend, and then the square root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736?

20736(144	144(12 root required.
1	1
24)107	22)44
96	44
284)1136	
1136	

*A GENERAL RULE**

FOR EXTRACTING THE ROOTS OF ALL POWERS.

1. Prepare the given number for extraction, by pointing off from the unit's place, as the required root directs.

* The extracting of roots of very high powers will, by this rule, be a tedious operation:—The following method, when practicable, will be much more convenient.

When the index of the power, whose root is to be extracted, is a composite number, take any two or more indices, whose produet is equal to the given index, and extract out of the given number a root answering to another of the indices, and so on to the last.

Thus, the fourth root=square root of the square root; the sixth root=square root of the cube root;—the eighth root=square root of the fourth root;—the ninth root=the cube root of the cube root;—the tenth root=square root of the fifth root;—the twelfth root=cube root of the fourth, &c.

2. Find the first figure of the root by trial, or by inspection into the Table of Powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the *dividend.*

4. Involve the root to the next inferior power to that which is given, and multiply it by the number, denoting the given power, for a *divisor.*

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the *given number* as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and, in like manner, proceed till the whole be finished.

EXAMPLES.

1. What is the cube root of 20346417?

$$\begin{array}{rl} & 20346417(273 \\ 2^3 = & 8 = \text{1st Subtrahend.} \\ \hline 2^2\times 3 = 12) & 123 = \text{Dividend.} \\ \hline 27^3 = & 19683 = \text{2d Subtrahend.} \\ 27^2\times 3 = 2187) & 6634 = \text{2d Dividend.} \\ \hline 273^3 = & 20346417 = \text{3d Subtrahend.} \\ \hline & \cdots\cdots \end{array}$$

$2\times 2\times 2=8$ root of the 1st period, or 1st subtrahend.
$2\times 2=4$(=next inferior power) and,
4×3 (the index of the given power)$=12=$1st divisor.
$27\times 27\times 27=19683=$2d subtrahend.
$27\times 27=729$ (=next inferior power) and,
729×3 (=index of the given power)$=2187=$2d div.
$273\times 273\times 273=27346417=$3d subtrahend.

2. What is the biquadrate root of 34827098976?

Ans. 431·9+.

3. Extract the square cubed, or sixth root of 1178420166615625. Ans. 325.

OF PROPORTION IN GENERAL.

NUMBERS are compared together, to discover the relations they have to each other.

There must be two numbers to form a comparison: The number, which is compared, being written first, is called the *antecedent;* and that, to which it is compared, the *consequent.*

Numbers are compared with each other two different ways: The one comparison considers the *difference* of the two numbers, and is called Arithmetical Relation, the difference being sometimes named the Arithmetical Ratio; and the other considers their *quotient*, which is termed Geometrical Relation, and the quotient, the Geometrical Ratio. Thus, of the numbers 12 and 4, the difference, or Arithmetical Ratio, is 12—4=8; and the Geometrical ratio is $\frac{12}{4}=3$.*

If two or more couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms, similarly posited, that is, either all the greater, or all the less taken as antecedents, and the rest as consequents, are called proportionals. So, the two couplets, 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.†

Proportion is distinguished into continued and discontinued. If, of several couplets of proportionals, written down in a series, the difference or ratio of each consequent, and

* Ratios are, here, always considered as the result of the greater term of comparison diminished, or divided by the less; not regarding which of them be the antecedent.

† Four numbers are said to be *reciprocally* or *inversely* proportional, when the fourth is less than the second by as many times as the third is greater than the first, or when the first is to the third, as the fourth to the second, and *vice versa.* Thus, 2, 9, 6, and 3 are reciprocal proportionals

Note. It is common to read the geometricals 2 : 4 :: 8 : 16, thus, 2 is to 4 as 8 to 16; or, as 2 to 4 so is 8 to 16.

the antecedent of the next following couplet, be the same as the common difference or ratio of the couplets, the proportion is said to be continued, and the numbers themselves, a series of continued arithmetical or geometrical proportionals. So 2, 4, 6, 8, form an arithmetical progression; for 4—2=6—4=8—6=2; and 2, 4, 8, 16, a geometrical progression; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

But if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinued. So 4, 2, 8, 6, are in discontinued arithmetical proportion; for 4—2=8—6=2= common difference of the couplets, 8—2=6=difference of the consequent of one couplet and the antecedent of the next; also, 4, 2, 16, 8, are in discontinued geometrical proportion; for $\frac{4}{2}=\frac{16}{8}=2$= commom ratio of the couplets, and $\frac{16}{2}=8$=ratio of the consequent of one couplet, and the antecedent of the next.

ARITHMETICAL PROPORTION.

THEOREM. If any four quantities, 2, 4, 6, 8, be in Arithmetical Proportion, the sum of the two means is equal to the sum of the two extremes.

And if any three quantities, 2, 4, 6, be in Arithmetical Proportion, the double of the mean is equal to the sum of the extremes.

THEOREM 2. In any continued arithmetical proportion, 1, 3, 5, 7, 9, 11, the sum of the two extremes, and that of every other two terms, equally distant from them, are equal: Thus, 1+11=3+9=5+7.

When the number of terms is odd, as in the proportion, 3, 8, 13, 18, 23, then, the sum of the two extremes, being double to the mean, or middle term, the sum of any other two terms equally remote from the extremes must likewise be double to the mean.

THEOREM 3. In any continued arithmetical proportion, 4, 4+2, 4+4, 4+6, 4+8, &c. the last or greatest term is equal to the first or least more the common difference of the terms drawn into the number of all the terms after the first, or into the whole number of the terms less one.

THEOREM 4. The sum of any rank, or series, of quantities in continued arithmetical proportion, 1, 3, 5, 7, 9, 11, is equal to the sum of the two extremes multiplied into half the number of terms.*

ARITHMETICAL PROGRESSION.

ANY rank of numbers, more than two, increasing by a common excess, or decreasing by a common difference, is said to be in Arithmetical Progression.

If the succeeding terms of a progression exceed each other, it is called an ascending series of progression; if the contrary, a descending series.

* The same thing also holds, when the number of terms is odd, as in the series, 4, 8, 12, 16, 20: for then the mean or middle term, being equal to half the sum of any two terms equally distant from it on contrary sides, it is obvious that the value of the whole series is the same as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the sum of the two extremes) multiplied by the whole number of terms; or to the sum of the extremes multiplied by half the number of terms.

The sum of any number of terms of the arithmetical series of odd numbers, 1, 3, 5, 7, 9, &c is equal to the square of that number.

For, 0+1 or the sum of 1 term=1^2 or 1
1+3 or the sum of 2 terms=2^2 or 4
4+5 or the sum of 3 terms=3^2 or 9
9+7 or the sum of 4 terms=4^2 or 16
16+9 or the sum of 5 terms=5^2 or 25, &c

EXAMPLES.

The first term, the ratio, and the number of terms given, to find the sum of the series.

A Gentleman travelled 29 days; the first day he went but 1 mile, and increased every day's travel 2 miles: How far did he travel?

29×29=841 miles the answer.

So { 0, 2, 4, 6, 8, 10, &c. is an ascending aritmetical series.
1, 2, 4, 8, 16, 32, &c. is an ascending geometrical series.

And { 10, 8, 6, 4, 2, 0, &c. is a descending arithmetical series
32, 16, 8, 4, 2, 1, &c. is a descending geometrical series.

The numbers, which form the series, are called the *terms* of the progression.

NOTE. The first and last terms of a progression are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be easily found.

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM I.

The first term, the last term, and the number of terms being given, to find the common difference.

RULE.

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

EXAMPLES.

1. The extremes are 3 and 39, and the number of terms is 19: What is the common difference?

$$\left.\begin{matrix}39\\-\ 3\end{matrix}\right\}\text{Ext.}$$

Divide by the number of terms—1=19—1=18)36(2 Ans.
18

Or, $\frac{39-3}{19-1}=2$

2. A man had 10 sons, whose several ages differed alike; the youngest was 3 years old, and the eldest 48; What was the common difference of their ages?

$$\frac{48-3}{10-1}=5 \text{ Ans.}$$

3. A man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing each day by an equal excess, so that the last day's journey may be 37 miles; required the daily increase.

$$\frac{37-5}{9-1}=4 \text{ Ans.}$$

PROBLEM II.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

RULE. Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. The extremes of an arithmetical series are 3 and 39, and the number of terms 19; required the sum of the series?

$$\left.\begin{matrix}39\\+\ 3\end{matrix}\right\}\text{Extremes.}$$

Sum=42

Number of terms=×19

378

42

2)798

399 Ans.

Or, $\frac{39+3\times19}{2}=399$

2. It is required to find how many strokes the hammer of a clock would strike in a week, or 168 hours, provided it increased 1 at each hour?

$$\frac{168+1\times168}{2}=14196 \text{ Answer.}$$

3. Suppose a number of stones were laid a yard distant from each other for the space of a mile, and the first a yard from a basket: what length of ground will that man travel

over who gathers them up singly, returning with them one by one to the basket?

$$\frac{3520+2\times1760}{2}=3099360 \text{ yards}=1761 \text{ miles, Ans.}$$

N. B. In this question, there being 1760 yards in a mile, and the man returning with each stone to the basket, his travel will be doubled; therefore the first term will be 2 and the last 1760×2, and the number of terms 1760.

4. A man bought 25 yards of linen in arithmetical progression; for the 4th yard he gave 12 cents, and for the last yard 75 cents. What did the whole amount to, and what did it average per yard?

$\frac{75-12}{22-1}=3$ the common difference, by which the first term is found to be 3.

Then $\frac{75+3\times25}{2}=9$D. 75c. and the average price is 39 cents per yard.

5. Required the sum of the first 1000 numbers in their natural order?

$$\frac{1000+1\times1000}{2}=500500 \text{ Ans.}$$

Problem III.

Given the extremes and the common difference, to find the number of terms.

Rule. Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the number of terms required.

Examples.

1. The extremes are 3 and 39, and the common difference 2: What is the number of terms?

39 } Extremes.
− 3 }

Common difference=2)36

Quotient=18

Add 1

19 Answer.

$$\text{Or, } \frac{39-3}{2} + 1 = 19.$$

2. A man going a journey travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles: How many days did he travel, and how far?

$$\frac{51-7}{4}+1=12 \text{ days, and } \frac{51+7\times12}{2}=348 \text{ miles, Ans.}$$

Problem IV.

The extremes and common difference given, to find the sum of the series.

Rule. Multiply the sum of the extremes by their difference increased by the common difference, and the product divided by twice the common difference will give the sum.

Examples.

1. If the extremes are 3 and 39, and the common difference 2, what is the sum of the series?

39+3=42=sum of the extremes.

39—3=36=difference of the extremes.

36+2=38=difference of the extremes increased by the common difference.

42
×38
336
126
1596

Twice the common difference=4)1596

399 Ans.

$$\text{Or, } \frac{39+3\times39-3+2}{2\times2}=399.$$

2. A owes B a certain sum, to be discharged in a year, by paying 6d. the first week, 18d. the second, and thus to increase every weekly payment by a shilling, till the last payment be £2 11s. 6d. what is the debt?

$$\frac{51{\cdot}5+{\cdot}5\times51{\cdot}5-{\cdot}5+1}{1\times2}=£67\ 12s. \text{ Answer.}$$

Problem V.

The extremes and sum of the series given, to find the common difference.

Rule. Divide the product of the sum and difference of the extremes by the difference of twice the sum of the series and the sum of the extremes, and the quotient will be the common difference.

Examples.

1. Let the extremes be 3 and 39, and the sum 399: what is the common difference?

Sum of the extremes=39+3= 42
Difference of the extremes=39—3= ×36

252
126

399×2—42=756)1512(2 Ans.
1512

Or, $\frac{39+2\times39-3}{399\times2-39+3}=2.$

2. A owes B £67 12s. to be discharged in a year, by weekly payments; the first payment to be 6d. and the last £2 11s. 6d.: What is the common difference of the payments, and what will each payment be?

$\frac{51{\cdot}5+{\cdot}5\times51{\cdot}5-{\cdot}5}{1352\times2-51{\cdot}5+{\cdot}5}$=1s. and 6d.+1s.=1s. 6d.=2d. payment, 1s. 6d+1s.=2s. 6d.=3d payment, &c.

Problem VI.

The extremes and sum of the series given, to find the number of terms.

Rule. Twice the sum of the series, divided by the sum of the extremes, will give the number of terms.

Examples.

1. Let the extremes be 3 and 39, and the sum of the series 399; what is the number of terms?

Sum of the series=399
× 2

Sum of extremes=39+3=42)798(19 Ans.
42

378
378

Or, $\frac{399\times2}{39+3}=19.$

2. A owes B £67 12s. to be paid weekly in arithmetical progression, the first payment to be 6d, and the last 51s. 6d.: How many payments will there be, and how long will he be in discharging the debt?

$\frac{1352\times2}{51\cdot5+\cdot5}=52$ payments, and as many weeks, Ans.

GEOMETRICAL PROPORTION.

THEOREM 1. If four quantities, 2, 6, 4, 12, be in geometrical proportion, the product of the means, 6×4, will be equal to that of the two extremes, 2×12, whether they are continued or discontinued; and, if three quantities, 2, 4, 8, the square of the mean is equal to the product of the two extremes.

THEOREM 2. If four quantities, 2, 6, 4, 12, are such that the product of two of them 2×12, is equal to the product of the other two, 6×4, then are those quantities proportional.

THEOREM 3. If four quantities, 2, 6, 4, 12, are proportional, the rectangle of the means divided by either extreme, will give the other extreme.

THEOREM 4. The products of the corresponding terms, of two geometrical proportions are also proportional.

That is, if 2 : 6 :: 4 : 12, and 2 : 4 :: 5 : 10, then will 2×2 : 6×4 :: 4×5 : 10×12.*

* And if any quantities be proportional, their squares, cubes, &c. will likewise be proportional.

THEOREM 5. If four quantities, 2, 6, 4, 12, are directly proportional,

Then,

1. Directly, $2 : 6 :: 4 : 12$
2. Inversely, $6 : 2 :: 12 : 4$
3. Alternately, $2 : 4 :: 6 : 12$
4. Compoundedly, $2 : 6+2 :: 4 : 12+4$
5. Dividedly, $2 : 6-2 :: 4 : 12-4$
6. Mixtly $6+2 : 6-2 :: 12+4 :: 12-4$
7. By Multiplication, $2r : 6r :: 4 : 12$
 or $2\times5 : 6\times5 :: 4 : 12$
8. By Division, $\frac{2}{r} : \frac{6}{r} :: 4 : 12$
 or, $\frac{2}{1} : \frac{6}{1} :: 4 : 12$

Because the product of the means in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 2.

THEOREM 6. If three numbers, 2, 4, 8, be in continued proportion, the square of the first will be to *that* of the second, as the first number to the third; that is, $2\times2 : 4\times4 :: 2 : 8$.

THEOREM 7. In any continued geometrical proportion, (1, 3, 9, 27, 81, &c.) the product of the two extremes, and *that* of every other two terms, equally distant from them, are equal.

THEOREM 8. The sum of any number of quantities, in continued geometrical proportion, is equal to the difference of the rectangle of the second and last terms, and the square of the first divided by the difference of the first and second terms.

GEOMETRICAL PROGRESSION.

A GEOMETRICAL Progression is when a *Rank*, or *Series*, of numbers, increases or decreases by the continual multiplication, or division, of some equal number.

PROBLEM I.

Given one of the extremes, the ratio, and the number of the terms of a geometrical series, to find the other extreme.

RULE. Multiply, or divide (as the case may require) the given extreme by such power of the ratio, whose ex-

ponent* is equal to the *number* of terms less 1, and the product or quotient, will be the other extreme.

EXAMPLES.

1. If the first term be 4, the ratio 4, and the number of terms 9, what is the last term?

* As the *last term*, or any term near the last, is very tedious to be found, by continual multiplication, it will often be necessary, in order to ascertain them, to have a series of numbers in arithmetical proportion, called *indices* or *exponents*, beginning with a cypher, or an unit, whose common difference is 1.

When the *first* term of the series and the *ratio* are *equal*, the *indices* must begin with an unit, and, in this case, the product of any two terms is equal to that term, signified by the *sum* of their *indices*.

Thus, { 1, 2, 3, 4, 5, 6, &c. indices or arithmetical series.
{ 2, 4, 8, 16, 32, 64, &c. geometrical series (leading terms)

Now 6+6=12=the index of the 12th term, and
64×64=4096=the twelfth term.

But when the *first* term of the series and the *ratio* are *different*, the *indices* must begin with a cypher, and the sum of the *indices*, made choice of, must be *one less* than the *number of terms*, given in the question; because 1 in the *indices* stands over the *second term*, and 2 in the *indices*, over the *third* term, &c. And in this case, the *product* of any *two* terms, divided by the *first*, is equal to that term *beyond* the first, signified by the *sum* of their *indices*.

Thus, { 0, 1, 2, 3, 4, 5, 6, &c. indices.
{ 1, 3, 9, 27, 81, 243, 729, &c. geometrical series.

Here, 6 + 5 = 11 the index of the 12th term
729×243=177147 the 12th term, because the first term of the series and the ratio are different, by which means a cypher stands over the first term.

Thus, by the help of these indices, and a few of the first terms in any geometrical series, any term, whose distance from the first term is assigned, though it were ever so remote, may be obtained without producing all the terms.

NOTE. If the *ratio* of any geometrical series be *double*, the *difference* of the *greatest* and *least* terms is equal to the sum of all the terms, except the greatest; if the ratio be *triple*, the *difference* is *double* the sum of all but the greatest; if the ratio be *quadruple*, the difference is *triple* the sum of all but the greatest, &c.

In any series in ÷ decreasing to infinity—if the *square* of the *first* term be divided by the *difference* between the *first* and *second*, the quotient will be the sum of the series.

1, 2, 3, 4+ 4= 8
4, 16, 64, 256×256=65536=power of the ratio, whose exponent is less by 1, than the number of terms.

65536=8th power of the ratio.
Multiply by 4=first term.

262144=last term.

Or $4\times4^8=262144$=the answer.

2. If the last term be 262144, the ratio 4, and the number of terms 9, what is the first term?

Last term.
8th power of the ratio $4^8=65536)262144(4$=first term.

Or, $\frac{262144}{4^8}=4$ the first term.

Again, Given the first term, and the ratio to find any other term assigned

RULE I.

When the indices begin with an unit.

1. Write down a few of the leading terms of the series, and place their indices over them.

2. Add together such indices, whose sum shall make up the entire index to the term required.

3. Multiply the terms of the geometrical series, belonging to those indices, together, and the product will be the term sought.

EXAMPLES.

1. If the first term be 2 and the ratio 2, what is the 13th term?

1, 2, 3, 4, 5+ 5+3= 13
2, 4, 8, 16, 32×32×8=8192 Answer.
Or, $2\times2^{12}=8192$.

2. A merchant wanting to purchase a cargo of horses for the West-Indies, a jockey told him he would take all the trouble and expense, upon himself, of collecting and purchasing 30 horses for the voyage, if he would give him what the last horse would come to by doubling the whole number by a half penny, that is, two farthings for the first, four for the second, eight for the third, &c. to which the merchant, thinking he has made a very good bargain, readily agreed:

Pray, what did the last horse come to, and what did the horses, one with another cost him?

1, 2, 3, 4, 5, 6+ 6=12th. 12+ 12+6=last ter.
2, 4, 8, 16, 32, 64×64=4096, & 4096×4096×64=
1073741824qrs,=£1118481 1s. 4d. and their average price was £37282 14s. 0½d. a piece.

Rule II.

When the indices begin with a cypher.

1. Write down a few of the leading terms of the series, as before, and place their indices over them.

2. Add together the most convenient indices to make an index, less by 1 than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series, together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power, whose index is one less than the number of terms multiplied, and make the result a divisor, by which divide the dividend, and the quotient will be that term beyond the first, signified by the sum of those indices, or the term sought.

3. If the first term be 5, and the ratio 3, what is the 7th term?

0, 1, 2, 3+ 2+ 1= 6=index to 6th term beyond
5, 15, 45, 135×45×15=91125=dividend. [the 1st or 7th.

The number of terms multiplied is 3, (viz. 135×35×15) and 3—1=2 is the power to which the term 5 is to be raised; but the second power of 5 is 5×5=25, and therefore 91125÷25=3645 the 7th term required.

Problem II.

Given the first term, the ratio, and number of terms, to find the sum of the series.

Rule. Raise the ratio to a power whose index shall be equal to the number of terms, from which subtract 1: divide the remainder by the ratio less 1 and the quotient, multiplied by the first term, will give the sum of the series.

EXAMPLES.

1. If the first term be 5, the ratio 3, and the number of terms 7, what is the sum of the series?

Ratio=3×3×3×3×3×3×3=2187=7th power of [the ratio.
Subtract 1

Divide by the ratio less 1=3—1=2)2186

Quotient=1093
Multiply by the first term= 5

Sum of the series=5465

$$\text{Or, } \frac{3^7-1}{3-1}\times 5=5465 \text{ Ans.}$$

2. What debt can be discharged in a year, by paying 1 cent the first month, 10 cents the second, and so on, each month in a tenfold proportion?

$$10^{12}-1\div 10-1\times 1=111111111111\text{c.}=$$
D.1111111111 11c. Ans.

3. A man threshed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the first day's work, 64 for the second, and so on in an eight fold proportion; I demand what his 9 day's labour amounted to, rating the wheat at 5s. per bushel?*

$$\frac{8^9-1}{8-1}\times 8=153391688 \text{ corns.}$$ Amount=£78 0s. 5½d.

4. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz. that he should give him a cent for the first door, 2 cents for the second, 4 cents for the third, and so on, doubling at every door, which were 36 in all; it is a bargain, cried the simpleton, and here is a guinea to bind it: Pray what did the house cost him?

$$\frac{2^{36}-1}{2-1}\times 1=68719476735\text{c.}=\text{D.}687194767\ 35\text{c.}\quad \text{Ans.}$$

* Note, 7680 wheat or barley corns are supposed to make a pint.

5. Suppose one farthing had been put out at 6 per cent per annum, compound interest,* at the commencement of the Christian æra: what would it have amounted to in 1784 years; and suppose the amount to be in standard gold, allowing a cubick inch to be worth £53 2s. 8d. how large would the mass have been?

$$\text{Ans.} \quad \frac{2^{1784}-1}{2-1} \times 1 =$$

£14867163465687482094357145515098907670653610 11s. 3¾d.
=279808597221212304159795712329 3594210766 cubick inches of gold.

As 355 : 113 :: $\overline{360\times69\cdot5}$: 7964 earth's diameter.

360×69·5×7964×1327·33=264482820122 cubick miles in the globe.
=672733373088547413688320 00 cub. inches in the globe.
Then, 279808597221212304159795712329 3594210766
÷672733373088547413688320 00
=415930899840288·8, which, however incredible it may appear to some, is more than four hundred and fifteen millions of millions, nine hundred and thirty thousand eight hundred and ninety nine millions, eight hundred and forty thousand two hundred and eighty eight times larger than the globe we inhabit.

Problem III.

The first term, the last term, (or the extremes) and the ratio given, to find the sum of the series.

Rule. Divide the difference of the extremes by the ratio less by 1: Add the greater extreme to the quotient, and the result will be the sum of all the terms.

Or, Multiply the greatest term by the ratio, from the product, subtract the least term; then divide the remainder by the ratio, less by 1, and the quotient will be the sum of all the terms.

* Any sum, at 6l. per cent. per annum, compound interest, will double in 11 years and three hundred and twenty-five days, or 11·889 years, or 11·89 is near enough; then if you divide 1784 by 11·89 it will give the number of terms in this case equal to 150:—The ratio will be 2, and the first term 1.

Or, When all the terms are given, then, from the product of the second and last term subtract the square of the first term; this remainder being divided by the second term less the first, will give the sum of the series.

EXAMPLES.

1. If the series be 2, 6, 18, 54, 162, 486, 1458, 4374, what is its total sum?

First Method.

From the greatest term=4374
Subtract the least = 2

Divide by the ratio—1=3—1=2)4372 diff. of extremes.

Quotient=2186
Add the greater extreme=4374

6560

Or, $\frac{4374-2}{3-1}+4374=6560$ Ans.

Second Method.

Greatest term=4374
Multiply by the ratio= 3

Product=13122
Subtract the least term= 2

Divide by the ratio less by 1=3—1=2)13120

6560 Ans.

Or, $\frac{4374\times3-2}{3-1}=6560.$

Third Method.

Greatest term=4374
Multiply by the second term= 6

Product=26244
Subtract the square of the first term=2×2= 4

Divide the remainder by the second *term* less the first. } =6—2=4)26240

Ans 6560

Or, $\frac{4374\times6-4}{6-2}=6560.$

2. A man travelled 6 days; the first day he went 4 miles, and each day doubling his day's travel, his last day's ride was 128 miles: How far did he go in the whole?

$$\frac{128-4}{2-1}+128=252 \text{ miles.}$$

3. A gentleman dying, left 5 sons; to whom he bequeathed his estate as follows, viz. to his youngest son £1000, to the eldest £5062 10s. and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$ What did the several legacies amount to?

$$\frac{5062{\cdot}5-1000}{1{\cdot}5-1}+5062{\cdot}5=£13187\ 10s.\ \text{Ans.}$$

Problem IV.

Given the extremes and ratio, to find the number of terms.

Rule.

Divide the greatest term by the least; find what power of the ratio is equal to the quotient: then, add 1 to the index of that power, and the sum will be the number of terms.

Or, Subtract the logarithm* of the least term from that of the greatest; divide the remainder by the logarithm of the ratio, and add 1 to the quotient.

Examples.

1. If the least term be 2, the greatest term 4374, and the ratio 3, what is the number of terms?

Divide by the least term=2)4374=the greatest term.

$3\times3\times3\times3\times3\times3\times3$=quotient, 2187=7th power of the ratio: then 7+1=8 Ans.

* Logarithms are artificial numbers, the addition of which answers to multiplication of whole numbers, and subtraction to division.

Or, from the logarithm of the greatest term = 3·64088
Subtract the logarithm of the least term = 0·30103
Divide the rema. by the logarithm of the ratio } = ·47712)3·33985(7+1=8, Answer.
3·33984

1

2. A gentleman travelled 252 miles: the first day he rode 4 miles, the last 128, and each day's journey was double to the preceding one: How many days was he in performing the journey? Ans. 6 days.

PROBLEM V.

Given the least term, the ratio, and the sum of the series, to find the last term.

RULE. Multiply the sum of the series by the ratio, less 1; to that product add the first term, and the result, divided by the ratio, will give the last term.

EXAMPLES.

If the 1st term be 2, the ratio 3, and the sum of the series 6560; what is the last term?

Sum of the series=6560
Multiply by the ratio less 1= 2

Product=13120
Add the least term= 2

Divide their sum by the ratio=3)13122

4374 Ans.

Or, $\frac{3-1\times6560+2}{3}=4374$ Ans.

2. A gentleman performed a journey of 252 miles; the first day he rode 4 miles, and each day after the first, twice so far as the day before: How far did he ride the last day?

$\frac{2-1\times252+4}{2}=128$ miles, Answer.

SIMPLE INTEREST,

INTEREST is a premium allowed by the borrower to the lender, according to a certain rate per cent. agreed on; which by law is stated at 6 per cent per annum. *Principal* is the money lent. *Rate* is the sum per cent. agreed on. *Amount* is the sum of principal and interest.

Simple Interest is that, which is allowed on the principal only.

NOTE. By this rule, Commission, Brokerage, Insurance, Purchasing Stocks, or any thing else, rated at so much per cent. are calculated.

GENERAL RULE.

1. Multiply the principal by the rate, and divide by 100, (or cut off the two right hand figures in the pounds) and the quotient, or left hand figures, will be the answer in pounds, &c. the right hand figures being reduced and cut off as at first. If the principal be dollars, the right hand figures will be cents.

2. For more years than one, multiply the interest of one year by the number of years.

3. For any number of months, take the aliquot parts of a year; and for days the aliquot parts of 30.

NOTE. When the rate per cent. per annum is { 9, 8, 6, 4, 3, 2 } multiply the principal by { $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ } of the given number of months, and you will have the interest for the given time.

EXAMPLES.

1. What is the interest of £573 13s. 9½d. at 6 per cent. per annum? Ans. £34 8s. 5d.

£573 13 9½
6

£34|42 2 9
20

8|42
12

5|13
4

|52

2. What is the interest of £329 17s. 6½d. for 3 years, 7 months, and 12 days, at 5 per cent. per annum?

£329 17 6½
5

16|49 7 8½
20

9|87
12

10|52
4

2|10

Then,

6 mo.	½	16	9	10½	interest of 1 year.
				3	
		49	9	7½	do. of 3 years.
1 mo.	1/6	8	4	11¼	do. of 6 months.
10 da.	⅓	1	7	5¾	do. of 1 month.
2 da.	1/5		9	1¾	do. of 10 days.
			1	9¾	do. of 2 days.

Ans. £59 13 do. of 3y. 7m. 12d.

Or thus:

£5	1/20	£	s.	d.
		329	17	6½
6 months	½	16	9	10½
				3
		49	9	7½
1 month	1/6	8	4	11¼
10 days	⅓	1	7	5¾
2 days	1/5		9	1¾
			1	9¾

£59 13s. Ans.

3. What is the interest of 347 dollars 50 cents, at 6 per cent. per annum for a year?

D347·50
6

20·8500 Ans. D.20 85c.

4. What is the interest of D.797 13c. at 6 per cent. per annum, for 8 months?

D797·13
4

31·8852 Ans. D31 88c. $5\frac{2}{10}$m.

5. What is the interest of D649 17c. at 6 per cent per annum, for 15 months?

D649 17
$7\frac{1}{2}$

454419
324585

48·68775 Ans. D48 68c· $7\frac{3}{4}$m.

6. Required the amount of £725 12s. 6d. at 5 per cent. per annum, for a year,

$5=\frac{1}{20}$)725 12 6

36 5 $7\frac{1}{2}$

Ans. £761 18 $1\frac{1}{2}$

7. What is the amount of D560 50c. at 6 per cent. for 16 months?

D560 50
8

44·84
560·50

Ans. D605·34

8. What is the interest of D150·75c. for 1 month, at 6 per cent. per annum?

½)150·75

·75375 Ans. 75c. 3¾m.

So that any number of dollars, considered as so many cents, is the interest for 2 months, at 6 per cent.

COMMISSION, OR FACTORAGE,

Is an allowance of so much per cent. to a Factor or Correspondent, for buying and selling goods.

9. Required the commission on £436 9s. 6d. at 3½ cent.

£	s	d
436	9	6
		3½
1309	8	6
218	4	9
15\|27	13	3
20		
5\|53		
12		
6\|39		
4		
1\|56		

Ans. £15 5 6¼.

10. Required the commission on D649 75c. at 1¾ per cent.

½	649·75
¼	324·875
	162·4375

1137·0625 Ans. D11 37c. 0½m.

BROKERAGE,

Is an allowance of so much per cent to a person called a Broker, for assisting merchants in purchasing or selling goods.

11. Required the brokerage on £911 12s. at 5s. or ¼ per cent.

5s.=¼)911 12

2|27 18
20

5|58
12

6|96
4

3|84

Ans. £2 5s. 6¾d.

12. Required the brokerage on D.876 21c. at 33⅓ cents, or at ⅓ per cent.

⅓)876·21

292·07 Ans. D.2 92c. $0\frac{7}{10}$m.

BUYING AND SELLING STOCKS.

STOCK is a general name for the capitals of trading companies.

13. Required the amount of £375 15s. bank stock, at £75 per cent.

| 50 | ½ | 375 15
| 25 | ½ | 187 17 6
93 18 9

Ans. £281 16 3

Or thus:

| 25 | ¼ | 375 15

Subtract 93 18 9

As before, £281 16 3

14. Required the amount of D.2195 50c. bank stock, at 125 per cent.

| 25 | ¼ | 2195·50
Add 548·875

Ans. D.2744·375.

SIMPLE INTEREST BY DECIMALS.

A TABLE of Ratios, from *one* pound to *ten* pounds.

Rate per cent.	Ratios.	Rate pr. ct.	Ratios.	Rate pr. ct.	Ratios.
1	·01	3	·03	5	·05
1¼	·0125	3¼	·0325	5½	·055
1½	·015	3½	·035	6	·06
1¾	·0175	3¾	·0375	6½	·065
2	·02	4	·04	7	·07
2¼	·0225	4¼	·0425	8	·08
2½	·025	4½	·045	9	·09
2¾	·0275	4¾	·0475	10	·1

Ratio is the Simple Interest of £1 or Dollar, for 1 year, at the rate per cent. agreed on.

A TABLE for the ready finding of the decimal parts of a year, equal to any number of *days*, or *quarters* of a year.

Days	dec. pts.	Days.	dec. pts.	Days	dec. pts.
1	·00274	10	·027397	100	·273973
2	·005479	20	·054794	200	·547945
3	·008219	30	·082192	300	·821918
4	·010959	40	·109589	365	1·000000
5	·013699	50	·136986		
6	·016438	60	·164383	¼ of a year = ·25	
7	·019178	70	·191781	½ of a year = ·5	
8	·021918	80	·219178	¾ of a year = ·75	
9	·024657	90	·246575		

CASE I.

The principal, time, and ratio given, to find the interest and amount.

RULE. Multiply the principal, time, and ratio continually together, and the last product will be the interest, commission, brokerage, &c. to which add the principal, and the sum will be the amount.

EXAMPLES.

1. Required the amount of £537 10s. at £6 per cent. per annum, for 5 years?

Principal 537·5
Multiply by the ratio= ·06

Product 32·250
Multiply by the time= 5

Interest=161·250
Add the principal=537·5

Amount=£698·75
20

15·00 Ans. £698 15s.

Or, 537·5×06×5+537·5=£698 15s.

2. What is the simple interest of £917 16s. at £5 per cent. per annum, for 7 years? Ans. £321 4s. 7d.

3. What is the amount of £235 3s. 9d. at £5¼ per cent. per annum, from March 5th, 1784, to November 23d, 1784?
Ans. £244 0 8½.

4. If my correspondent is to have £2½ per cent. what will his commission on £785 15s. amount to?
Ans. £19 12 10½.

5. If a broker disposes of a cargo for me, to the amount of £637 10s. on commission at £1¼ per cent. and procures me another cargo of the value of £817 15s. on commission at 1¾ per cent.; what will his commission on both cargoes amount to? Ans. £22 5 7.

CASE II.

The amount, time, and ratio given, to find the principal.

RULE.

Multiply the ratio by the time; add unity to the product, for a divisor, by which sum divide the amount, and the quotient will be the principal.

EXAMPLES.

1. What principal will amount to £1045 14s. in 7 years, at £6 per cent. per annum?

Ratio=·06
Multiply by the time= 7

Product=·42
Add 1·

Divisor=1·42)1045·7(736·4084 + = £736 8s. 2d. answer.

Or, $\frac{1045 \cdot 7}{\cdot 06 \times 7 + 1.}$ = £736 8s. 2d. Ans.

2. What principal will amount to £3810 in 6 years, at £4½ per cent. per annum? Ans. £3000.

3. What principal will amount to £666 9s. 0½d. in 3¼ years, at £5½ per cent. per annum? Ans. £563.

CASE III.

The amount, principal, and time given, to find the ratio.

RULE.

Subtract the principal from the amount; divide the remainder by the product of the time and principal, and the quotient will be the ratio.

EXAMPLES.

1. At what rate per cent. will £543 amount to £705 18s in 5 years?

From the amount=705·9
Take the principal=543

Divide by 543×5=2715)162·90(·06
162·90

Or, $\frac{705 \cdot 9 - 543}{543 \times 5}$ = ·06 = £6 Answer.

2. At what rate per cent. will £391 17s. amount to £449 3s. 1¾d. ·74qr. in 3¼ years? Ans. £4½.

3. At what rate per cent. will £413 12s. 6d. amount to £546 4s. 10½d. in 4¾ years? Ans. £6¾.

CASE IV.

The amount, principal, and rate per cent. given, to find the time.

RULE.

Subtract the principal from the amount; divide the remainder by the product of the ratio and principal, and the quotient will be the time.

EXAMPLES.

1. In what time will £543 amount to £705 18s. at £6 per cent. per annum?

From the amount=705·9
Take the principal=543

Divide by 543×·06=32·58)162·90(5 years, Ans.
162·90

2. In what time will £3000 amount to £3810, at £4½ per cent. per annum? Ans. 6 years.

3. In what time will £391 17s. amount to £449 3s. 1½d. at £4½ per cent. per annum? Ans. 3¼ years.

To find the interest of any sum at 6 per cent. per annum, for any number of months.

RULE.

If the months be an even number, multiply the principal by half that number; and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus, the half of 19 is 9·5; and multiply the principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first.

4. What is the interest of £345 16s. 6d. for 9 years and 11 months, at 6 per cent. per annum?

Y. m.
9 11
12

2)119 months.

59·5=½ number of months.

Principal 345.825
Mult. by ½ number of Mo. - 59·5

1729125
3112425
1729125

£205·765875=£205 15 3¾ Ans.
Principal=£345 16 6

Amount=£551 11 9¾

A TABLE of decimal parts for every day in the twelfth part of a year, which consists of 365¼ days.

D.	dec.	D.	dec.	D.	dec.	D.	dec.	D.	dec.
1	·033	7	·230	13	·427	19	·624	25	·821
2	·066	8	·263	14	·460	20	·657	26	·854
3	·098	9	·296	15	·493	21	·690	27	·887
4	·131	10	·328	16	·526	22	·723	28	·920
5	·164	11	·361	17	·558	23	·756	29	·953
6	·197	12	·394	18	·591	24	·788	30	·986

NOTE. This table may also be used for the *parts of a year*, in Compound Interest after having worked for whole years.

To find the Interest of any sum, either for months, or months and days, at 6 per cent. per annum.

RULE.

Multiply the principal by the number of months, (or months and parts, answering to the given number of days in the table) and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the interest for the given time, in shillings and decimal parts of a shilling.

EXAMPLES.

1. What is the interest of £100 for a year?

Principal=100
Multiply by the month= 12

Ans. s.120|0=£6.

2. What is the interest of £250 10s. for 19 months and 7 days?

Principal=£250·5
Time= 19·23

7515
5010
22545
2505

Ans. s.481·7115=£24 1 8½.

Another method of calculating Interest for Months, at £6 per cent per annum.

RULE.

If the principal consist of pounds only, cut off the unit figure, and, as it then stands, it will be the interest for 1 month in shillings and decimal parts:—If it consist of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a shilling.

EXAMPLES.

1. What is the interest of £175 for 5 months?

£175=17·5s.=interest for 1 month. 17·5
Multiply by the time= 5

2|0)87·5

Ans. £4 7 6

2. What is the interest of £255 16s. for 7 months?

£255 16s.=25·58 int. for 1 shil. [month.
7

2|0)179·06

£8 19 0½ Ans.

SIMPLE INTEREST IN FEDERAL MONEY.

PROBLEM I.

When the principal is given in Massachusetts pounds, shillings, &c. and the interest is required in Federal Money, at 6 per cent. per annum.

RULE.

Reduce the shillings, &c. to their equivalent decimal, divide the whole by 5, and the quotient is the annual interest: Or, multiply the principal by 2, and the product

(having the unit figure of the pounds cut off,) will be the interest as before.

EXAMPLES.

1. Required the annual interest of £517 3s. 7½d. at 6 per cent.

3s.=·15
7½d.=·030
Excess of 12 =·001
·181

5)517·181
103·436=103 43 6 Ans.

Or, 517·181
2
103·4362=D.103 43s. 6$\frac{2}{10}$m.

2. Required the annual interest of £1 in cents.

5)1·00
20 cents, Ans.

PROBLEM II.

When the principal is given in Massachusetts, old currency, and the interest and amount are required in Federal Money, at 6 per cent.

RULE.

Reduce the Massachusetts money to Federal, then divide the principal by 20, and that quotient by 5; add those quotients together, and they are the interest; or add them to the principal, and they are the amount.

EXAMPLES.

1. Required the amount of £425 16s. 8½d. for 1 year, at 6 per cent.

·8
·034
·001
16s. 8d.=£·835

3)425·835
20)1419·450
5)70·9725
14·1945
1504·6170=1504 61 7 Ans.

2. Required the amount of £112 4s. 6d. for a year.

·2	3)112·225
·024	20)374·083
·001	5)18·7041
	3·7408
4s. 6d. = £·225	D. c. m. dec.
	396·5279 = 396·52 7 9 Ans.

PROBLEM III.

When the principal is Massachusetts old currency, and the monthly interest is required in Federal Money.

RULE.

Reduce the shillings, &c. to decimals, then separate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the answer in dollars, cents, &c.

EXAMPLE.

Required the monthly interest of 425l. 16s. 8½d. in Federal money?

·8	6)42·5835
·034	D. c. m.
·001	7·09725 = 7·09 7½ Ans.
£·835	

PROBLEM IV.

When the principal is Federal Money, and the interest is required in the same.

RULE.

Work according to the general rule in simple interest, that is, multiply by the rate of interest, separate the two right hand figures of the dollars in the product, and it will give the interest in dollars, cents, &c.

N. B. The figures, which are more than three places to the right hand of the point, are of no account, unless the fourth place exceed 5, in which case increase the mills 1.

EXAMPLES.

1. What is the annual interest of D.537 24c. 6m. at 6 per cent.?

537·24 6
6

D. c. m.
32·23476 = 32·23 5 Ans.

2. What is the interest of D.1465 46c. 6m. for 18 months, at 6 per cent. per annum?

1465·46 6

8=half the number of months.

D. c. m.

117·23728=117·23 7 Ans.

3. What is the interest of D.537 34c. 7m. for 19 months, at 6 per cent. per annum?

537·34 7

9·5=half the number of months.

2686735
4836123

D. c. m.

51·047965 = 51·04 8 Ans.

N. B. Because there are 4 decimals in the multiplicand and multiplier, I cut off 4 figures for them, and two more according to the rule.

PROBLEM V.

When the principal is Federal Money, and the monthly interest is required in the same, at 6 per cent. per annum.

RULE.

Separate the two right hand figures of the dollars, and you then have the interest for two months; half of which is the monthly interest in dollars, cents, &c. If there be but one place, or figure of dollars, a cypher must be prefixed to the left hand.

EXAMPLES.

1. What is the monthly interest of D.9 59c. 7m. at 6 per cent. per annum? 2)·09597

Ans. ·047985=4c. 8m. nearly.

2. What is the monthly interest of D.100 50c. 5m.? 2)1·00505

·50252=50c. 2½m. Ans.

RULES *for calculating Interest for Days.*

RULE I.

Multiply the principal by the given number of days, and that product by the rate on the pound; divide the last product by 365 (the number of days in a year) and it will give the interest.

EXAMPLE.

What is the interest of £360 10s. for 175 days, at 6 per cent.?

$$\frac{360{\cdot}5\times175\times{\cdot}06}{365}=£10{\cdot}37=£10\ 7s.\ 4\tfrac{4}{5}d.\ \text{Answer.}$$

RULE II.

Multiply the given principal by the given number of days, and divide the product by 6083, for 6 per cent.; (the number of days in which any sum will double at that rate) the quotient will give the answer.

EXAMPLE.

What is the interest of £327 10s. at 6 per cent. per annum, for 210 days?

$$\frac{327{\cdot}5\times210}{6083}=£11{\cdot}306=£11\ 6\ 1\tfrac{1}{4}\ \text{Ans.}$$

RULE *for making a divisor for any rate per cent.*

Multiply 365 by 100, and divide the product by the rate.

Thus for 6 per cent. $\frac{365\times100}{6}=6083$ divisor

For 5 per cent. $\frac{365\times100}{5}=7300$ divisor, &c.

Perhaps the most convenient way to calculate at 6 per cent. is first to do it for 5, and then add one fifth of the quotient to itself; because by cutting off the two cyphers in the divisor, you have to divide only by 73.

Hence, when interest is to be calculated on cash accounts, accounts current, or any other accounts where partial payments are made, or partial debts contracted; multiply the several balances into the days they are at interest, and the sum of these products, divided as above, will give the interest at 5l. or 6l. per cent, and for any

other rate, make the proper addition or deduction; or find a divisor, as before directed.

Examples.

1. On the 1st of January I lent £450 10s. 6d. which I received back in the following partial payments. viz. on the 14th of January £57 11s. 9d.; on the 7th of February, £39 3s. 10d.; on the 19th of March, £63 5s. 2d.; on the 4th of April, £45; on the 26th of April, £19 12s. 6d.; on the 12th of May, £100; on the 10th of June, £60 7s. 3d.; and on the 1st. of August, £65 10s.: What interest is due at 6 per cent.?

Dates.		£	s.	d.	Da.	Products.		
Jan. 1	Lent on demand	450	10	6	13	5856	16	6
14	Received in part	55	11	9				
	Balance	392	18	9	24	9430	10	0
Feb. 7	Received in part	39	3	10				
	Balance	353	14	11	40	14149	16	8
Mar. 19	Received in part	63	5	2				
	Balance	290	9	9	16	4647	16	0
April 4	Received in part	45	0	0				
	Balance	245	9	9	22	5409	14	6
26	Received in part	19	12	6				
	Balance	225	17	3	16	3613	16	0
May 12	Received in part	100	0	0				
	Balance	125	17	3	29	3650	0	3
June 10	Received in part	60	7	3				
	Balance	65	10	0	52	3406	0	0
Aug. 1	Received in full of the principal	65	10	0		50155	9	11

```
                                  5)
73|00)501|55   9 11(6   17   4¼  interest at 5 per cent.
      438          1    7   5¼
      ---         ---
       6355        £8   4  10¼  interest at 6 per cent.
         20
      ----
     )1271|09
        73
      ----
        541
        511
      ----
       3009
         12
      ----
     )361|19
        292
      ----
       6919
          4
      ----
     )276|76
        219
      ----
       5776
```

2. I have given Peter Trusty a cash credit for 1000l. in consequence of which, on the 12th of May, I paid his bill for 250l.; May 27th, paid his draught for 280l.; June 1st, he gave me a bill on the Massachusetts bank at sight, for 290l.; July 17th, he paid me per receipt, 70l.; August 20th, he drew for 750l. at sight; August 31, he paid me per receipt, 500l.; Sept. 15, he drew at sight for 135l.; and 3d of October for 175l.; Oct. 29th, he paid me, per receipt, 250l.; and Nov. 3d, 125l.; November 12th, he drew at sight for 375l.; and November 18th, for 125l.; January 1st, he paid me, per receipt, 290l.; and January 20th, 210l. On the 1st of March following, he demanded a settlement: What is then due to me, interest at 6 per cent.?

Dates.			£	Days.	Prod.
May	12	Paid his bill	250	15	3750
May	27	Paid his draught	280		
		Balance	530	5	2650
June	1	Received in part	290		
		Balance	240	46	11040
July	17	Received in part	70		
		Balance	170	34	5780
August	20	Paid	750		
		Balance	920	11	10120
August	31	Received in part	500		
		Balance	420	15	6300
Sept.	15	Paid	135		
		Balance	555	18	9990
October	3	Paid	175		
		Balance	730	26	18980
October	29	Received in part	250		
		Balance	480	5	2400
November	3	Received in part	125		
		Balance	355	9	3195
Nov.	12	Paid	375		
		Balance	730	6	4380
Nov.	18	Paid	125		
		Balance	855	44	37620
January	1	Received in part	290		
		Balance	565	19	10735
January	20	Received in part	210		
March	1	Balance	355	40	14200
					141140

5)
Then, 73|00)141140(19 6 8 interest at 5 per cent.
3 17 4

£23 4 0 interest at 6 per cent.
355 0 0

£378 4 0 balance in my favour.

When cash credits are given, a balance should be made upon every transaction, which should be multiplied into the days the first leisure minute; then, when the time of settlement comes, you will only have to add up the products, and divide as above, and the account will be finished.

3. A owes B the following sums, with the interest on them, at 6 per cent. per annum, as follows, viz. D.60 for 7 months, D.150 for 15 months, D.75 50c. for 9 months, D.145 75c. for 27 months, and D.397 60c. for 45½ months: What is the amount of principal and interest?

D.c.	Mo.		
60	× 7	=	420
150	×15	=	2250
75·5	× 9	=	679·5
145·75	×27	=	3935·25
397·60	×45·5	=	18090·8
828.85			

200)25375·55(126·877 interest.
828·85 principal.

D.955·727 amount, Ans.

NOTE. I divide by 200, the number of months, in which any sum will double at 6 per cent. per annum, and it gives the interest.

When partial payments are made upon bonds, notes, &c. at any interval *greater* than a year, the interest is calculated in a progressive manner, by adding the interest to the principal at the time of the first payment, and from the sum deducting the payment, &c.

DISCOUNT

IS an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The *present worth* of any sum or debt, due some time hence, is such a sum, as, if put to interest, would in that time and at the rate per cent. for which the discount is to be made, amount to the sum or debt then due.

RULE.

As the amount of 100l. for the given rate and time, is to 100l. : so is the given sum or debt to the present worth.

Subtract the present worth from the given sum, and the remainder will be the discount required.

Or, As the amount of 100l. for the given rate and time, is to the interest of 100l. for that time: so is the given sum or debt to the discount required.

Or, *In Federal Money*, divide the given sum by the amount of D.100 for the given time and rate; point off from the right of the quotient two places less than in division of decimals, for the present worth.

EXAMPLES.

1. What is the discount of { £635 17s. / D.2119 50c. } due two years hence, at 5½ per cent. per annum?

Interest of 100l. per annum=5 10
2

11
Add 100

111

As £111 : £11 :: 635l. 17s. : 63l. 0s. 2¾d. disc. Ans.

Or,

£ £ £ s. £ s. d.
As 111 : 100 :: 635 17 : 572 16 9¼ present worth.

And 635l. 17s.—572l. 16s. 9¼d.=63l. 0. 2¾d. discount.

In Federal Money.

D. D. D. c. D. c. m.
As 111 : 11 :: 2119 50 : 210 04 0½=discount. Or,

$$\text{As } 111 : 100 :: 2119\cdot50 : \frac{2119\cdot5\times100}{111} = \text{D.}1909\ 45\text{c.}$$

9½m.=present worth; and 2119·5—1909·4595=210·0405 =discount, as before.

Or, $\frac{2119\cdot5}{111}$=19·094595; and 1909·4095=present worth, as before.

2. What is the present worth of D.350 payable in half a year, discounting at 6 per cent. per annum?

Ans. D.339 80c. 5m.

3. What is the present worth of 65l. due 15 months hence, at 6l. per cent. per annum? Ans. 60l. 9s. 3½d.

4. What is the discount on 97l. 10s. due January 22, this being September 7, reckoning interest at 5l. per cent.?

Ans. £1 15s. 11d.

5. Bought a quantity of goods for D.250 ready money, and sold them for D.300 payable 9 months hence: What was the gain in ready money, supposing discount to be made at 6 per cent.? Ans. D.37 8c. 1¼m.

BARTER

IS the exchanging of one commodity for another, and teaches traders to proportion their quantities without loss.

CASE I.

When the quantity of one commodity is given, with its value, or that of its integer, that is, of 1*lb.* 1*cwt.* 1*yd, &c. as also the value of the integer of some other commodity, to be given for it, to find the quantity of this; or having the quantity thereof given, to find the rate of selling it.*

Rule.

Find the value of the given quantity by the concisest method, then find what quantity of the other, at the rate proposed, you may have for the same money: Or, if the

quantity be given, find from thence the rate of selling it. *Or*, As the quantity of one article is to its price, so, *inversely*, is the quantity of the other to its price. *Or*, as the price of one article is to its quantity; so *inversely*, is the price of the other to its quantity.

Examples.

1. How much tea, at 9s. 6d. per lb. must be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon?

3d.	$\frac{1}{4}$	Gals. 156
$\frac{1}{2}$	$\frac{1}{6}$	12
		1872
		39
		6 6
9s. 6d.=114d.		1917 6
		12
		23010

d. lb. d. lb. oz.
As 114 : 1 :: 23010 : 201 $13\frac{5}{114}$ Ans.

price. quan. price quan.
Or, As 12s. $3\frac{1}{2}$d. : 156gal. :: 9s. 6d. : 201 lb. $13\frac{5}{114}$oz.
answer, as before.

2. How much cloth at 15s. 8d. per yard, must be given for 5cwt. 3qrs. 19lb. of steel, at 5 guineas per cwt.?
Ans. 52yds. 3qrs. 2n.

3. Suppose A has 350 yards of linen, at 25c. per yard, which he would truck with B for sugar, at D.5 per cwt. How much will the linen come to?
Ans: 17cwt. 2qrs.

4. A has broadcloths at D.44 per piece, and B has mace, at D.1 42c. per lb.: How many pounds of mace must B give A for 35 pieces of cloth? Ans. $1084\frac{1}{2}$lbs.

5. A has $7\frac{1}{2}$cwt. of sugar at 12 cents per lb. for which B gave him $12\frac{1}{2}$cwt of flour: What was the flour rated at per lb.? Ans. 7c. 2m.

CASE II.

If the quantities of two commodities be given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.

RULE.

Find the separate values of the two given commodities; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quality and rate may be easily found.

EXAMPLES.

1. Two merchants barter; A has 30cwt of cheese, at 23s. 6d. per cwt. and B has 9 pieces of broadcloth, at 31. 15s. per piece: Which must receive money, and how much? Ans. B must pay A £1 10s.

2. A and B would barter; A has 150 bushels of wheat, at D.1 25c. per bushel, for which B. gives 65 bushels of barley, worth 62½c. per bushel, and the balance in oats at 37½c. per bushel: What quantity of oats must A receive from B? Ans. 391⅓ bushels.

CASE III.

Sometimes, in bartering, one commodity is rated above the ready money price; then, to find the bartering price of the other, say,

As the ready money price of the one, is to its bartering price; so is *that* of the other, to *its* bartering price: Next, find the quantity required, according to either the bartering or ready money price.

EXAMPLES.

1. A has ribbands at 2s. per yard ready money; but in barter he will have 2s. 3d.; B has broadcloth at 32s. 6d. per yard ready money; at what rate must B value his cloth per yard, to be equivalent to A's bartering price, and how many yards of ribband, at 2s. 3d. per yard, must then be given by A for 488 yards of B's broadcloth?

Ans. B's broadcloth, at £1 16s. 6½d. per yd.; 7930 yds. ribband.

2. A and B barter; A has 150 gallons of brandy, at D.1 37½c. per gallon ready money, but in barter he will have D.1 50c.: B has linen at 44c. per yard ready money; how must B sell his linen per yard in proportion to A's bartering price, and how many yards are equal to A's brandy?

Ans. Barter price is 48c. and he must give A 468¾yds.

3. P and Q barter: P has Irish linen, at 60c. per yard, but in barter he will have 64c. Q delivers him broadcloth at D.6 per yard, worth only D.5 50c. per yard: Pray which had the advantage in barter, and how much linen does P give Q for 148 yards of broadcloth?

c. c. D. c. D. c.
As 60 : 64 :: 5 59- : 5 86⅔; therefore Q, by selling at D.6, has the advantage. Then,

D. yds. c. yds. qrs.
As 6 : 148 :: 64 : 1387 2 linen, Ans.

4. A has 200 yards of linen, at 1s. 6d. ready money per yard, which he barters with B. at 1s. 9d. per yard, taking buttons at 7½d. per gross, which are worth but 6d.: How many gross of buttons will pay for the linen, who gets the best bargain, and by how much, both in the whole, and per cent.?

yd. d. yds. d. d. gross. d. gross.
As 1 : 21 :: 200 : 4200. As 7½ : 1 :: 4200 : 560

yd. d. yds. £.
As 1 : 18 :: 200 : 15 value of A's linen.

gr. d. gr. £
As 1 : 6 :: 560 : 14 value of B's goods. So that B gains £1 of A.

£ £ £
As 14 1 : 100 : £7 2s. 10d. per cent.

LOSS AND GAIN

IS an excellent rule, by which merchants and traders discover their profit, or loss per cent, or by the gross: It also instructs them to raise or fall the price of their goods, so as to gain or lose so much per cent, &c.

CASE I.

To know what is gained or lost per cent.

RULE.

First see what the gain or loss is by subtraction; then, as the price it cost, is to the gain or loss, so is 100l. to the gain or loss per cent, &c.

Or, in *federal money*, annex two cyphers to the gain or loss, and divide by the cost for the gain or loss per cent.

EXAMPLES.

1. If I buy serge at 90c. per yard, and sell it again at D.1 2c. per yard: What do I gain per cent. or in laying out D.100?

Sold for D.1·02 — As 90 : 12 :: 100 : $13\frac{1}{3}$ per cent.
Cost ·90 — gain, Ans.

Gain ·12 per yard.

Or, 1·02—·90=·12=gain per yard; and $\frac{12\cdot00}{\cdot90}=13\frac{1}{3}$ per cent. gain, as before.

N. B. The first questions in the several cases serve to elucidate each other.

2. If I buy serge at D.1 2c. per yard, and sell it again at 90c.: What do I lose per cent. or in laying out D.100?

Cost D.1·02 — D. c. D. D. c. m.
Sold for ·90 — As 1·02 : 12 : 100 : 11 76 5 per cent. loss, Ans.
Loss ·12

Or, $\frac{12\,00}{1\cdot02}=11\cdot215$ per cent. loss, Ans. as before.

3. If I buy a cwt. of tobacco for 9l. 6s. 8d. and sell it again at 1s, 10d. per lb. do I gain or lose, and what per cent.?

	£	s.	d.
Sold for	10	5	4
Cost	9	6	8
	0	18	8 gained in the gross.

	£	s.	
2d \| $\frac{1}{12}$ \|	11	4	value at 2s. per lb.
	0	18	8 value at 2d. per lb.
	10	5	4 value at 1s. 10d. per lb.

	£	s.	d.	s.	d.	£	£	
As	9	6	8	: 18	8	:: 100	: 10	Ans. 10 per cent. gain.

4. A draper bought 60 yards of cloth at D.4 50c. per yard, and 38 yards of cloth at D.2 50c. per yard, and sold them at D.4 25c. per yard: Did he gain or lose, and what per cent.?

60 yards at	D.4 50c. per yard	=	D.270
38 yards at	2 50 per yard	=	95
98 yards cost	- - - - -		365
which subtract from 98 yards at D.4 25c.		=	416·50
	Gained in the gross	=	51·50

Then, as 365 : 51·50 :: 100 : $\frac{5150\cdot00}{365}$ = 14·11 gain per cent. Ans.

5. Bought sugar at 6½d per lb. and sold it at 2l. 3s. 9d per cwt.: What was the gain or loss per cent.?

lb. d. lb. £ s. d.
As 1 : 6½ : 112 : 3 0 8

Prime cost	£3	0	8 per cwt.
Sold at	2	3	9 per cwt.
Lost	£0	16	11 in the whole.

£ s. d. s. d. £ £ s. d.
As 3 0 8 : 16 11 :: 100 : 27 17 8¼ loss per cent. Ans.

6. At 4s. 6d. in the pound profit, how much per cent.?

£ s. d. £ £ s.
As 1 : 4 6 :: 100 : 22 10 Ans.

7. If I buy candles at 1s. 6d. per lb. and sell them again at 2s. per lb. and allow three months for payment, what do I gain per cent.?

d. d. £ £ s. d.
As 18 : 24 :: 100 133 6 8;

Mo. £ Mo. £
Then, by Discount, As 12 : 6 :: 3 : 1 10s.

£ s. £ s. £ s. d. £ s. d.
Then, As 101 10 : 1 10 :: 133 6 8 : 1 10 4¾
which taken from £133 6s. 8d. leaves £131 7s. 3¼d. therefore, Ans. £31 7s. 3¼d.

8. If I buy cloth at 13s. per yard, on 8 months credit, and sell it again at 12s. ready money, do I gain or lose, and what per cent. ?

Mo. £ Mo. £. £ s. £ s. d.
As 12 : 6 :: 8 : 4 As 104 : 13 :: 100 : 12 6;

So that 13s. on 8 months credit, at 6 per cent. is equal to 12s. 6d. ready money; then,

Prime cost	12s.	6d.	ready money.
Sold at	12	0	ready money.
Lost		6	in the yard.

As 12s. 6d. : 6d. :: £100 : £4 Ans. £4 per cent.

CASE II.

To know how a commodity must be sold, to gain or lose so much per cent.

RULE. As 100l. is to the price: so is 100l. with the profit added, or loss subtracted, to the gaining or losing price.

Or, In *Federal Money*, multiply 100 dollars added to the gain, or less by the loss per cent. by the cost; and pointing off the two right hand figures of the product gives the answer.

EXAMPLES.

If I buy a quantity of serge, at 90c. per yard: How must I sell it per yard to gain $13\frac{1}{3}$ per cent. ?

D. D. c. c. D. c.
As 100 : 113 $33\frac{1}{3}$: 90 : 1 2 Ans.

Or, D.113 $33\frac{1}{3}$c.×90c.=D.102; and pointing off two right hand places, D.1·02, Ans. as before.

2. If a barrel of powder cost 4l. how must it be sold to lose 10 per cent. ?

£ £ £
As 100 : 4 :: 90

4

100)360(3
300

60
20

100)1200(12 Ans. £3 12

1200

Or thus:

90
4

£3|60
20

s. 12|00

In casting up the amount of goods bought, imported or exported; to the prime cost of such goods we must add all the charges upon them, in order to fix the price they stand us in.

3. Suppose I import from France, 12 bales of cloth, containing 10 pieces each, which, with the charges there, amounted to D.360: I pay duty here 92c. per piece, for freight D.12, and porterage D.1 25c.: What does it stand me in per piece, and how must I sell it per piece to gain D.10 per cent.?

	D. c.
12 bales	First cost 360·
10	Duty 110·40
120 pieces	Freight 12·
	Porterage 1·25
	Whole cost 483·65

Pieces.	D. c.	Piece	D. c.	
As 120	: 483·65	:: 1	: 4·03	cost per piece

	D.	per ct.	D. c.	c. m.	
Again, as	100	: 10	:: 4·03	: 40 3	gain.
			40 3		

D.4·43 3 the price at which it must be sold per piece.

4. Bought cloth, at D.2 50c. per yard, which not proving so good as I expected, I am content to lose 17½ per cent. by it: How must I sell it per yard?

Ans. D.2 6c. 2½m.

5, If 120lb of steel cost 7l. how must I sell it per lb. to gain 15½l. per cent.?

As 120lb. : £7 :: 1lb. : 1s. 2d.
As £100 : 1s. 2d. :: £115½ : 1s. 4d. per lb. Ans.

6. Bought fish in Newburyport, at 10s. per quintal, and sold it at Philadelphia, at 17s. 6d. per quintal; now allowing the charges at an average, or one with another, to be 2s. 6d. per quintal, and considering I must lose 20l. per cent. by remitting my money home: what do I gain per cent.?

Selling price 17 6 Philadelphia currency, per quintal.
Charges 2 6 ditto.

15 0 ditto.

£ s. £ s.
As 100 : 15 :: 80 : 12 New-England currency.

Sold at 12s. per quintal.
Bought at 10s. per quintal.

Gained 2s. per quintal.

s. s. £ £
As 10 : 2 :: 100 : 20 per cent. gained, Ans.

7. Bought 50 gallons of brandy, at 75c. per gallon, but by accident, 10 gallons leaked out: At what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of 10 per cent.? Ans. D1 3c. 1¼m.

CASE III.

When there is gain or loss per cent. to know what the commodity cost.

RULE. As 100l. with the gain per cent. added, or loss per cent. subtracted, is to the price; so is 100l. to the prime cost.

Or, In *Federal Money*, divide the price with two cyphers annexed, by D.100 added to the gain, or less by the loss, per cent, for the answer.

EXAMPLES.

1. If 1 yard of cloth be sold, at D.1 2c. and there is gained 13⅓ per cent. what did the yard cost?

D. c. D. c.
As D.100+13⅓ : 1 2 : 100 : 90 prime cost, Answer.

Or, $\frac{102{\cdot}00}{113{\cdot}33\frac{1}{3}} = 90$ Ans. as before.

2. If 12 yards of cloth are sold at 15s. per yard, and there is £7 10s. loss per cent. in the sale: What is the prime cost of the whole?

Yd. s. yds. £
As 1 : 15 :: 12 : 9

£ s. £ £ £ s. d.
As 92 10 : 9 :: 100 : 9 14 7 Ans.

3. If 19½cwt. sugar be sold at D.14 50c. per cwt. and I gain D.15 per cent. What did it cost per cwt.?

D.		D. c.		D.	D.
As 115	:	14·50	::	100	: 12·608 Ans.

CASE IV.

If by wares sold at such a rate, there is so much gained or lost per cent. to know what would be gained or lost per cent. if sold at another rate.

RULE.

As the first price is to 100l. with the profit per cent. added, or loss per cent subtracted; so is the other price, to the gain or loss per cent. at the other rate.

N. B. If your answer exceed 100, the excess is your gain per cent.; but if it be less than 100, the deficiency is your loss per cent.

EXAMPLES.

1. If cloth sold at D.1 2c. per yard, be 13½ profit per cent. what gain or loss per cent. shall I have, if I sell the same at 90c. per yard?

D. c.		D.		c.		D.
As 1 2	:	113½	::	90	:	100

And 100—100=0 Ans. I neither gain nor lose.

2. If cloth sold at 4s. per yard, be 10l. per cent. profit: What shall I gain or lose per cent. if sold at 3s. 6d. per yard?

s.		£		s. d.
As 4	:	110	:	3 6
12				12
48				42
				110

48)4620(96¼ Ans. I lost 3¾l. per cent. by the last sale.
432

300
288

12

Then, 100l.—96¼l.=3¾l.

3. If I sell a gallon of wine for D.1 50c. and thereby lose 12 per cent.: What shall I gain or lose per cent. if I sell 4 gallons of the same wine for D.6 75c.?

D. c. D. c. c.

As 6 : 88 :: 6 75 : 99 And 100—99=1, per cent. loss.

EQUATION OF PAYMENTS

IS the finding a time to pay at once several debts due at different times, so that no loss shall be sustained by either party.

RULE I.*

Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time, or that required.

EXAMPLES.

1. A owes B D.380 to be paid as follows, viz. D.100 in 6 months, D.120 in 7 months, and D.160 in 10 months; What is the equated time for the payment of the whole debt?

100× 6= 600
120× 7= 840
160×10=1600

100+120+160=380)3040(8 months, Ans.
3040

2. A owes B £104 15s. to be paid in 4½ months, £161 to be paid in 3½ months, and £152 5s. to be paid in 5 months: What is the equated time for the payment of the whole?

Ans. 4 months and 8 days.

* This rule is founded upon the supposition that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms.

3. There is owing to a merchant £698 to be paid, £178 ready money, £200 at 3 months, and £320 in 8 months; I demand the indifferent time for the payment of the whole?

Ans 4½ months.

4. The sum of D.164 16c. 6m. is to be paid, ½ in 6 months, ⅓ in 3 months and ⅙ in 12 months: What is the mean time for the payment of the whole?

Ans. 7⅔ months.

Rule II.

See by Rule 1, at what time the first man, mentioned, ought to pay in his whole money; then as his money is to his time, so is the other's money, to his time, inversely, which when found, must be added to, or subtracted from, the time at which the second ought to have paid in his money, as the case may require, and the sum, or remainder, will be the time of the second's payment.

Examples.

1. A is indebted to B D.150, to be paid, D.50 at 4 months, and D.100 at 8 months: B owes A D.250, to be paid at 10 months: It is agreed between them that A shall make present pay of his whole debt, and that B shall pay his so much the sooner, as to balance that favour: I demand the time at which B must pay the D.250, reckoning simple interest.

$$50\times4=200$$
$$100\times8=800$$

$$50+100=150)100|0(6\tfrac{2}{3} \text{ months, A's equated time.}$$
$$90$$
$$10$$

	D.	mo.		D.	mo.
As	150	6⅔	::	200	4

mo. mo. mo.

Then, 10—4=6 time of B's payment.

2. A merchant has D.1200 due to him, to be paid ¼ at 2 months, ¼ at 3 months, and the rest at 6 months; but the debtor agrees to pay ½ down: How long may the debtor detain the other half, so that neither party may sustain loss?

mo. mo.
$\frac{1}{4} \times 2 = 0\frac{1}{2}$
$\frac{1}{4} \times 3 = 1$
$\frac{1}{2} \times 6 = 3$

Equated time $= 4\frac{1}{2}$

Now, as $\frac{1}{2}$ was paid $4\frac{1}{2}$ months before it was due, it is reasonable that he should detain the other $\frac{1}{2}$ $4\frac{1}{2}$ months after it became due, which, added, gives $8\frac{1}{2}$ months, the true time for the second payment.

POLICIES OF INSURANCE.

INSURANCE is a security, or assurance, by means of a writ called a *Policy*, to indemnify the insured of such losses as shall be specified in the policy subscribed by the insurer, or insurers, by which the underwriters oblige themselves to make good and effectual the property insured, in consideration of a certain premium at a stipulated rate per cent. (which varies according to the risk) to be immediately paid down, or otherwise secured according to the tenor of the agreement.

The average loss is 10 per cent.; that is, if the insured suffer any damage or loss, not exceeding 10 per cent. he bears it himself, and the insurers are free.

A policy should be taken out for a sum sufficient to cover the principal and premium, and the business of this rule is, in general, to calculate what that sum should be.

CASE I.

When the premium, at a certain rate per cent. for insuring a sum, is required, the operation is the same as in interest, or commission.

EXAMPLES.

1. What is the premium upon 537l. 15s. 9d. at $6\frac{1}{2}$ per cent.?

£	s.	d.
537	15	9
		6½
3226	14	6
½= 268	17	10½
34\|95	12	4½
20		
19\|12		
12		
1\|48		
4		
1\|94		

Ans. 34l. 19s. 1¼d. nearly.

2. What is the premium upon D.375, at 7½ per cent. ?

D.375
·075
1875
2625
Ans. D.28·125

CASE II.

To find the sum for which a policy should be taken out to cover a given sum.

RULE.

Take the premium from 100l. (or in *federal money* D.100) and say, As the remainder is to 100l. so is the sum adventured to the policy.* Or,

* Now it is plain, that if I want to recover 92l. I must in this case, insure upon 100l. : therefore to recover 759l. I must insure upon 825l. ; for when 8 per cent. for premium is deducted, I shall have 759l. remaining nett.

For £825=sum insured upon at 8 per cent.
66=premium to be deducted.
759=the first outset.

In *decimals*, take the premium from 100, annex two cyphers to the sum to be covered, and divide by the remainder for the policy.

EXAMPLES.

1. It is required to cover 759l. premium 8 per cent.: For what sum must the policy be taken?

100
8
—
92 : 100 :: 759
100
—
92)75900(825l. Ans.
736
—
230
184
—
460
460

Or, $\frac{75900}{92}$ = 825l. Ans. as before.

2. A merchant sent a vessel and cargo to sea, valued at D.5760: What sum must the policy be taken out for, to cover this property, premium 19½ per cent.?

100
19·5
—
80·5 : 100 :: 5760 : 7155·28—Ans.
D. D. D. c.
576000

Or, $\frac{576000}{80·5}$ = D.7155·28c.—Ans. as before.

CASE III.

When a policy is taken out for a certain sum, in order to cover a given sum.

RULE.

To find the premium, say, As the policy is to the covered sum; so is £100 (or D.100) to a fourth number, which, being taken from 100, will leave the premium.

Or, In *decimals*, divide the sum covered, with two cyphers annexed, by the policy; subtract the quotient from 100, and the remainder is the premium.

EXAMPLES.

1. If a policy be taken out for 1250l. to cover 500l. what is the premium per cent.?

1250 : 500 :: 100
100

1250)50000(40 and 100l.—40=60l. Ans.

Or, $\frac{50000}{1250}$=40, &c. as before.

2. If a policy be taken out for D.781·25, to cover D.625: Required the premium per cent.

D. D. D. D. c.
As 781·25 : 625 :: 100 : 87 50 And 100—87·5=12·5, or 12½ per cent. premium, Ans.

Or, $\frac{62500}{781·25}$=87·5, &c. as before.

CASE IV.

When the policy for covering any sum and the premium per cent. are given, to find the sum to be covered.

RULE.

Deduct the premium per cent, from 100, and say, As 100 is to the remainder, so is the policy to the sum required to be covered. Or,

In *decimals*, multiply the policy by the remainder found as before, and point off two right hand places in the product for the answer.

EXAMPLES.

1. If a policy be taken out for £1250 at 60 per cent. what is the adventure, or sum to be covered?

100
60

100 : 40 :: 1250
40

100)50000(500l. Ans.

Or, $\overline{1250\times100-60}=50000$, and, pointing off two places, 500·00 Answer, as before.

2. If a policy be taken out for D.781 25c. at 12½ per cent. required the sum covered?

$$\text{As } 100 : 100-12\tfrac{1}{2} :: 781\cdot25 : \frac{781\cdot25\times100-12\tfrac{1}{2}}{100} =$$

D.625 Answer.

Or, $781\cdot25\times100-12\cdot5=62500$; and 625·00 Answer as before.

COMPOUND INTEREST

IS that which arises from the interest being added to the principal, and (continuing in the hands of the borrower) becoming a part of the principal, at the end of each stated time of payment.

Method I.

Rule.*

Find the *amount* of the given principal, for the time of the first payment, by Simple Interest: next, find the interest of that sum, or principal, and add it as before, and thus proceed for any number of years, still accounting the last amount as the principal for the next payment. The given principal being subtracted from the last amount, the remainder will be the compound interest.

In *federal money*, multiply the principal by the rate for the first time of payment, setting the product two places more to the right than the multiplicand, and the decimal point in the product under that in the multiplicand; then find the amount, and proceed as above.

* It may be observed that all computations, relating to Compound Interest, are founded upon a series of terms, increasing in Geometrical Progression, wherein the number of years assigns the index of the last and highest term: Therefore, as one pound is to the amount of one pound, for any given time, so is any proposed principal, or sum, to its amount for the same time.

NOTE. It is not usually necessary to carry the work beyond mills; therefore, when the figure next beyond mills at the right, exceeds 5, increase the number of mills 1 when it does not exceed 5, it may be omitted. The result will be exact enough for common purposes.

EXAMPLES.

1. What will $\left\{\begin{matrix}£480\\D.1600\end{matrix}\right\}$ amount to in 5 years, at 6 per cent. per annum?

Principal £480
Rate of interest 6

28|80
20

16|00

Principal for the first year	£480	0
Interest of ditto	28	16
Principal for the second year	508	16
		6
	30\|52	16
	20	
	10\|56	
	12	
	6\|72	
	4	
	2\|88	

Principal for the second year	£508	16	0
Interest for ditto	30	10	6¼
Principal for the third year	539	6	6¼
			6
	32\|35	19	3
	20		
	7\|19		
	12		
	2\|31		
	4		
	1\|24		

Principal for the 3d year	£539	6	6¼
Interest for ditto	32	7	2¼
Principal for the 4th year	571	13	8½
			6
	34\|30	2	4½
	20		
	6\|02		
	12		
	0\|28		
	4		
	1\|14		

Principal for the 4th year	£571	13	8½
Interest for do.	34	6	0¼
Principal for the 5th year	605	19	9
			6
	36\|35	18	6
	20		
	7\|18		
	12		
	2\|22		

Principal for the 5th year	£605	19	9
Interest for ditto	36	7	2
Amount for 5 years	642	6	11
Subtract the first principal	480	0	0
Compound interest for 5 years	162	6	11

In Federal Money thus:

Principal for the first year	D.1600·	
Rate of interest	·6	
Interest first year	96·00	
Amount 1st and principal 2d year	1696·	

Carried over.

Brought over	1696· 6
Interest 2d year	101·76
Amount 2d year, principal 3d	1797·76 6
Interest 3d year	107·8656
Amount 3d year, principal 4th	1905·6256 6
Interest 4th year	114·337536
Amount 4th year, principal 5th	2019·963136 6
Interest 5th year	121·19778816
Amount for 5 years	2141·16092416
Subtract first principal	1600·
Compound interest for 5 years =	541·16092416

Or thus:

	D.
First principal	1600. 6
Interest	96·00
Second principal	1696· 6
Interest	101·76
Third principal	1797·76 6
Interest	107·866
Fourth principal	1905·626

Fourth principal	D.1905·626 6
Interest	114·338
Fifth principal	2019·964 6
Interest	121·198
Amount	2141·162
1st principal	1600·
Compound interest	541.162 nearly, as before.

2. What is the compound interest of D.740 for 6 years, at 4 per cent. per annum? Ans. D.196 33c. 1m.

3. What will $\left\{\begin{matrix}£150\\D.500\end{matrix}\right\}$ amount to in a year, at 2 per cent. a month? Ans. $\left\{\begin{matrix}£190\ 4s.\ 5d.\\D.634\ 12c.\ 1m.\end{matrix}\right\}$

Method II.

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner find the amount for every succeeding year.

When the rate is at 6 per cent. per annum.

1. Divide the principal by 20, and that quotient by 5: these quotients, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner obtain the amount for every succeeding year.

EXAMPLES.

1. What is the amount of 480l. at 6 per cent. per annum, for 5 years?

20)480
5) 24
4 16

20)508 16 amo. of 1st year.
5) 25 8 $9\frac{1}{2}$
5 1 9

20)539 6 $6\frac{1}{2}$ ditto of 2d.
5) 26 19 $3\frac{3}{4}$
5 7 $10\frac{1}{4}$

20)571 13 $8\frac{1}{2}$ ditto of 3d.
5) 28 11 $8\frac{1}{4}$
5 14 4

20)605 19 $8\frac{3}{4}$ ditto of 4th.
5) 30 5 $11\frac{3}{4}$
6 1 $2\frac{1}{4}$

£642 6 $10\frac{3}{4}$ do. 5th, Ans.

2. Of the same sum at 5 per cent. for 5 years?

20)480
24

20)504 amount of 1st. yr.
25 4

20)529 4 ditto of 2d.
26 9 $2\frac{1}{4}$

20)555 13 $2\frac{1}{4}$ ditto of 3d.
26 15 $7\frac{3}{4}$

20)583 8 10 do. of 4th.
29 3 $5\frac{1}{4}$

£612 12 $3\frac{1}{4}$ do. of 5th, Ans.

NOTE. The same may be done in Federal Money, but the first method is generally more easy.

COMPOUND INTEREST BY DECIMALS.

A TABLE of the amount of £1 or D.1, at $\frac{1}{2}$ per cent. per month, as practised at the Banks.

Mon.	£ or D. dec. pts.	Mon.	£ or D. dec. pts.	Mon.	£ or D. dec. pts.
1	1·005	5	1·025	9	1·045
2	1·01	6	1·03	10	1·05
3	1·015	7	1·035	11	1·055
4	1·02	8	1·04	12	1·06

A TABLE of the amount of £1 or D.1, from 1 Day to 31 days, at 6 per cent. per annum.

Days.	£ or D. dec. pts.	Days.	£ or D. dec. pts.	Days.	£ or D. dec. pts.
1	1·00016	12	1·00197	22	1·00361
2	1·00032	13	1·00213	23	1·00378
3	1·00049	14	1·0023	24	1·00394
4	1·00065	15	1·00246	25	1·0041
5	1·00082	16	1·00263	26	1·00427
6	1 00098	17	1·00279	27	1·00443
7	1·00115	18	1·00295	28	1·0046
8	1·00131	19	1·00312	29	1·00476
9	1·00147	20	1·00328	30	1·00493
10	1·00164	21	1·00345	31	1·00509
11	1·00180				

CASE I.

When the principal, the rate of interest, and time, are given, to find either the amount or interest.

RULE.

1. Find the amount of £1 or D.1 for one year at the given rate per cent.

2. Involve the amount, thus found, to such power, as is denoted by the number of years; or, in Table I, at the end of Annuities, under the rate and against the given number of years, you will find the power.

3. Multiply this power by the principal, or given sum, and the product will be the amount required, from which, if you subtract the principal, the remainder will be the interest.

EXAMPLES.

1. What is the Compound interest of 600l. for 4 years, at 6 per cent. per annum?

1·06= { amount of £1 for 1 year, at 6 per cent. per ann.

Multiply by 1.06

1·1236=2d power.

Multiply by 1·1236

1·26247696=4th power.

Multiply by 600=principal.

757·48617600=amount.

Subtract 600

157·486176=£157 9s. 8½d. interest required.

By TABLE I.

Tabular amount of £1 for 4 years, at 6 per cent. per annum, =1·2624769

Multiply by the principal= 600

Amount=757·4861400

2. What is the amount of D.1500 for 12 years at 3½ per cent per annum?

D.1·035=amount of D.1 for 1 year at 3½ per cent. per annum.

And $1{\cdot}035^{12} \times 1500$=D.2266 60c. nearly, Ans.

ANOTHER METHOD

Of working compound interest for *years*, *months*, and *days*, which is much more concise than the preceding method.

RULE.

To the logarithm of the principal, found in any Table of logarithms, add the several logarithms, answering to the number of *years*, *months*, and *days*, found in the following tables, and their sum will be the logarithm of the amount for the given time, which being found in any table of logarithms, the natural number corresponding thereto will be the answer.

LOGARITHMICK TABLES, at 6 per cent. per annum, for Years, Months, and Days.

Years.	dec. pts.	Y.	dec. pts.	Months.	dec. pts.
1	·025306	21	·531426	1	·002166
2	·050612	22	·556732	2	·004321
3	·075918	23	·582038	3	·006466
4	·101224	24	·607344	4	·0086
5	·12653	25	·63265	5	·010724
6	·151836	26	·657956	6	·012837
7	·177142	27	·683262	7	·01494
8	·202448	28	·708568	8	·017033
9	·227754	29	·733974	9	·019116
10	·25306	30	·75938	10	·021189
11	·278366	31	·784586	11	·023252
12	·303672	32	·809792		
13	·328978	33	·835098		
14	·354284	34	·860404		
15	·37969	35	·88571		
16	·404896	36	·911016		
17	·430202	37	·936322		
18	·455508	38	·961628		
19	·480814	39	·986934		
20	·50612	40	1·01224		
Days.		D.		Days.	
1	·000071	12	·000857	22	·001568
2	·000143	13	·000928	23	·001639
3	·000215	14	·000999	24	·00171
4	·000287	15	·00107	25	·001781
5	·000358	16	·001142	26	·001852
6	·000429	17	·001213	27	·001923
7	·0005	18	·001284	28	·001994
8	·000571	19	·001355	29	·002065
9	·000642	20	·001426	30	·002136
10	·000713	21	·001497	31	·002207
11	·000785				

What is the amount of £132 10s. at 6 per cent. per annum, for 9 years, 8 months and 15 days?

	To the log. of £132·5	= 2·122216
Add	Log. for 9 years	= ·227754
	ditto for 8 months	= ·017033
	do. for 15 days	= ·00107
		2·368073
Because 8 months are past, deduct 4 per cent. upon the log. of 15 days		= ·0000428
	Remains	2·3680302

The nearest to which, in the table of logarithms, is 2·368101, and the natural number answering thereto is 233·4=£233 8s. Ans.

CASE II.

When the amount, rate and time, are given, to find the principal.

RULE.

Divide the amount by the amount of £1 or D.1 for the given time, and the quotient will be the principal.

Or, If you multiply the present value of £1 or D.1 for the given number of years, at the given rate per cent. by the amount, the product will be the principal, or present worth.

EXAMPLES.

1. What is the present worth of £757 9s. 8½d. due 4 years hence, discounting at the rate of £6 per cent. per annum?

BY TABLE I.

Divide by the tabular am. of £1 for 4 years = 1·2624769)757·4861400(£600 Ans.

BY TABLE II.

Amount=757·48614

Mult. by the present worth of £1 for 4 years, at 6 per cent. per an. = ·7920936

Ans. 599·999923582704+=£600

2. What principal must be put to interest 6 years, to amount to D.689·42140338094531 25, at 5½ per cent. per annum? Ans. D.500

CASE III.

When the principal, rate, and amount, are given, to find the time.

RULE.

Divide the amount by the principal; then divide this quotient by the amount of £1 or D.1 for 1 year, this quotient by the same, till nothing remain, and the number of the divisions will show the time.

Or, Divide the amount by the principal, and the quotient will be the amount of £1 or D.1 for the given time, which seek under the given rate in Table 1, and in a line with it, you will see the time.

EXAMPLE.

In what time will D.500 amount to D.689 42c. 1m.+, at 5½ per cent. per annum?

500	689·421+
1·055	1·379—
1·055	1·307—
1·055	1·239—
1·055	1·174+
1·055	1·113—
1·055	1·053+
	1·—

6 divisions.

Ans. 6 years.

CASE IV.

When the principal, amount and time, are given, to find the rate per cent.

RULE.

Divide the amount by the principal, and the quotient will be the amount of £1 or D. for the given time; then, extract such root as the time denotes, and that root will be the amount of £1 or D.1 for one year, from which subtract unity, and the remainder will be the ratio.

Or, Having found the amount of £1. or D.1 for the time as above directed, look for it in Table 1st, even with the

given time, and directly over the amount you will find the ratio.

EXAMPLE.

At what rate per cent. per annum will D.500 amount to D.689·421403+ in 6 years?

$$\frac{689\cdot421403+}{500}=1\cdot378843-;\text{ and }\sqrt[6]{1\cdot378843-}=1\cdot055.$$

Then 1·055—1=·055=ratio. Hence the rate is 5½ per cent. per annum, Answer.

DISCOUNT BY COMPOUND INTEREST.

The sum, or debt to be discounted, the time and rate, given, to find the present worth.

RULE.

Divide the debt by that power of the amount of £1 or D.1 for 1 year, denoted by the time, and the quotient will be the present worth, which subtracted from the debt, will leave the discount.

EXAMPLES.

1. What is the present worth, and discount, of 600l. due 3 years hence at 6l. per cent. per annum, compound interest?

Divide by $\overline{1\cdot06}^{3}=1\cdot19101$) 600·00000 (503·7741 = £503 15s. 5¾d. present worth, and £600—£503 15s. 5¾d. = £96 4s. 6¼d. = discount.

Or, $\frac{600}{1\cdot19101}=£503\cdot7741$ and $600-\frac{600}{1\cdot19101}=$ £96·2259.

By TABLE II.

In this Table, corresponding to the time and rate, we

·839619 = { have present worth of £1 for the time and rate.

Multiply by 600 = debt, or principal.

503·771400 = present worth of the debt.

2. What ready money will discharge a debt of D.1000 due 4 years hence, at 5 per cent. per annum, compound interest? Ans. D.822 70c. 2m.

ANNUITIES OR PENSIONS IN ARREARS, AT COMPOUND INTEREST.

CASE I.

When the annuity, or pension, the time it continues, and the rate per cent. are given to find the amount.

RULE.

1. Make the first term of a Geometrical Progression, and the amount of £1 or D.1 for 1 year at the given rate per cent. the ratio.

2. Carry the series to so many terms as the number of years, and find its sum.

3. Multiply the sum thus found by the given annuity, and the product will be the amount sought.

Or, multiply the amount of £1 or D.1 for 1 year into itself so many times as there are years less by 1; then multiply this product by the annuity; and subtract the annuity therefrom. Lastly, divide the remainder by the ratio less 1, and the quotient will be the amount.

EXAMPLES.

1. What will an annuity of 60l. per annum, payable yearly, amount to in 4 years, at £6 per cent.?

First Method.

$1+1\cdot06+\overline{1\cdot06}^{2}+\overline{1\cdot06}^{3}=4\cdot374616=$sum.

Multiply by $60=$annuity

$262\cdot476960$

20

$9\cdot53920$

12

$6\cdot4704$

4

$1\cdot8816$ Ans. £262 9s. 6¼d.

Or $1+1\cdot06+\overline{1\cdot06}^{2}+\overline{1\cdot06}^{3}\times60=$£262 9s. 6¼d.

Second Method.

1·06×1·06×1·06×1·06=1·26247

Multiply by 60 annuity.

75·74820

Subtract 60

Divide by 1·06—1=·06)15·7482(262·47=£262 9s. 4$\frac{3}{4}$d.

12

37
36

14
12

28
24

42
42

Or, $\frac{1\cdot06\times1\cdot06\times1\cdot06\times1\cdot06\times60-60}{1\cdot06-1}=£262\cdot47$

Or, by TABLE III.

Multiply the tabular number under the rate and opposite to the time by the annuity, and the product will be the amount.

2. What will an annuity of £60 per annum amount to in 20 years, allowing £6 per cent. compound interest?

Under £6 per cent. and opposite 20, in Table 3, you will find

Tabular number=36·78559

Multiply by 60=annuity.

2207·13540=£2207 2s. 8$\frac{1}{4}$d. Ans.

3. What will wages, of D.25 per month, amount to in a year, at $\frac{1}{2}$ per cent. per month?

Ans. D.308 38c. 9m.

CASE II.

When the amount, rate per cent. and time are given, to find the annuity, pension, &c.

RULE.

Multiply the whole amount by the amount of £1 or D.1 for a year, from which subtract the whole amount, divide the remainder by that power of the amount of £1 or D.1 for a year, signified by the number of years, made less by unity, and the quotient will be the answer.

Or, Find the amount of an annuity of £1 or D.1 for the given time and rate (by Case I); divide the given sum by this amount; and the quotient will be the annuity required.

EXAMPLES.

1. What annuity being forborne 4 years, will amount to £262·47696, at 6 per cent. compound interest?

```
                1·06
                1·06
                ----
                 636
                1060
                ----
              1·1236
                1·06
                ----
               67416
              112360
              ------
             1·191016
                1·06
                ----
             7146096
            11910160
            --------
            1·26247696
Subtract    1·
            --------
            ·26247696=Divisor.
```

```
                262·47696=amount.
Mult. by        1·06=am. of £1
                ---------- for 1 yr.
               157486176
               262476960
               ---------
              278·2255776
Subt.         262·47696
              ---------
·26247696)15·7486176(60l. Ans.
          15·7486176
          ----------
                   0
```

$$\text{Or, } \frac{262{\cdot}47696\times1{\cdot}06-262{\cdot}47696}{1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06-1}=60.$$

Or thus.

Amount of an annuity of £1 for 4 years, at 6 per cent. per annum,=4·374616 (by Case I); and

$$\frac{262{\cdot}47696}{4{\cdot}374616} = £60 \text{ Ans.}$$

Or, by Table III. the amount of £1 is found to be 4·374616; and the answer is found as before.

2. What annuity, being forborne 20 years, will amount to D.2207·1354, at 6 per cent. compound interest?

Amount of an annuity of D.1 for 20 years at 6 per cent. per annum=36·78559: And

36·78559)2207·13540(D.60, Ans.
2207·1354

0

CASE III.

When the annuity, amount and ratio, are given to find the time.

RULE.

Multiply the amount by the ratio, to this product add the annuity, and from the sum subtract the amount; this remainder being divided by the annuity, the quotient will be that power of the ratio signified by the time, which being divided by the amount of £1 for one year, and this *quotient* by the same, till nothing remain, the number of those divisions will be equal to the time. Or, look for this number under the given rate in Table I, and in a line with it, you will see the time. Or,

Divide the amount by the annuity; from the quotient subtract 1: from the remainder subtract the ratio; from successive remainders, subtract the square, cube, &c. of the ratio, till nothing remain; and the whole number of the subtractions will be the answer. Or, find the quotient in Table III. under the rate, and in a line with it stands the answer.

EXAMPLES.

1. In what time will £60 per annum, payable yearly, amount to £262·47696, allowing £6 per cent. compound interest for the forbearance of payment?

262·47696=amount.
Multiply by 1·06=ratio.

157486176
262476960

278·2255776
Add 60· =annuity.

338·2255776
Subtract 262·47696

Divide by 60)75·7486176.

Divide by 1·06)1·26247696

Divide by 1·06)1·191016

Divide by 1·06)1·1236

Divide by 1·06)1·06

1 Ans. 4 years.

The number of Divisions, by 1·06, being 4, gives the number of years=4, the answer.

Or thus:

Annuity=60)262·47696=amount.

4·374616
1. Subtract 1·

3·374616
2. Subtract 1·06 =ratio.

2·314616
3. Subtract 1·1236 = ratio.^{2}

1·191016
4. Subtract 1·191016= ratio.^{3}

. Answer, 4 years.

Or, looking into Table III. under the rate, 6, the quotient, 374616, stands against 4 years, answer, as before.

Or, in Table I. under the given rate, you will find 262476, and in a line, under years, you will find 4.

PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

CASE I.

When the annuity, &c. rate and time are given, to find the present worth.

RULE.*

1. Divide the annuity by the ratio, or the amount of D.1 £1 for 1 year, and the quotient will be the present worth 1 year's annuity.
2. Divide the annuity by the square of the ratio, and e quotient will be the present worth for 2 years.
3. In like manner, find the present worth of each year r itself, and the sum of all these will be the present value the annuity sought.

Or, divide the annuity, &c. by that power of the ratio nified by the number of years, and subtract the quotient om the annuity; this remainder being divided by the tio less 1, the quotient will be the present worth.

EXAMPLES.

1. † What ready money will purchase an annuity of 60l. continue 4 years, at 6l. per cent. Compound interest?

* The *amount* of an annuity may also be found for years and rts of a year, thus:
1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a ar.
3. Add this interest to the former amount, and it will give the ole amount required.
The *present worth* of an annuity for years and parts of a year ay be found thus:
1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for e given parts of a year, and it will be the whole present worth quired.

† Questions in this case may also be answered by first finding the ount of the given annuity by Case I. of annuities in arrears, p. 9, and then the present worth, or principal, by Case II. of Com- *und Interest*, page 246.

First Method.

Ratio = 1·06)60·00000(56.603=present worth for [1 year.

$\overline{\text{Ratio}}^2$ = 1·1236)60·00000(53·399= do. for 2 years.

$\overline{\text{Ratio}}^3$ = 1·191016)60·00000(50·377= do. for 3 years.

$\overline{\text{Ratio}}^4$ =1·26247696)60·00000(47·525= do. for 4 years.

Ans. 207·904=£207 18s. 0¾d.

Second Method.

4th power of the ratio } =1·26247696)60·0000000(47·525

From 60

Subtract 47·525

Divisor=1·06—1=·06)12·475

207·916=£207 18s. 3¾d.

Or, $\frac{60}{\cdot 06}$ =47·525 60—47·525=12·475

And $\frac{12{\cdot}475}{\cdot 06}$ =207·916.

By Table III.

Under £6 per cent. and opposite 4 we find

4·37461=amount of £1, annuity for 4 years.

Multiply by 60=annuity.

262·47660=amount of £60 for 4 years.

Then opposite 4 years and under £6 per cent. in Table 2,

We have ·792093

Multiply by 262·7466

4752558
4752558
3168372
5544651
1584186
4752558
1584186

208·1197426338=£208 2s. 4½d.

Or, opposite 4 years, and under £6 per cent. in Table 1st, we have 1·26247=the amount of £1 for 4 years.

Then, 262·7466÷1·26247=208·1209=£208 2s. 5d.

By Table IV.

Multiply the tabular number, under the rate, and opposite the time, into the annuity, and the product will be the present worth.

Thus, in Example 1st. What ready money will purchase £60 annuity, to continue 4 years, at £6 per cent. compound interest?

Under £6 per cent. and even with 4 years,

We have 3·4651=present worth of £1 for 4 years.

Multiply by 60=annuity.

Ans.=207·9060=£207 18s. 1¼d.

2. What is the present worth of an annuity of D.60 per annum, to continue 20 years, at 6 per cent. compound interest? Ans. D.688·65 (nearly.)

CASE II.

When the present worth, time and rate are given, to find the annuity, rent, &c.

Rule.

1. From that power of the ratio, denoted by the number of years, plus 1, subtract that power of it denoted by the number of years.

2. Divide the remainder by that power of the ratio, signified by the time made less by unity.

3. Multiply the present worth into this quotient, and the product will be the annuity, pension, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the present worth.

2. Multiply that power of the ratio, denoted by the time, by the present worth, and subtract this product from the former.

3. Divide the remainder by that power of the ratio, denoted by the time made less by unity, and the quotient will be the annuity.

Examples.

1. What annuity, to continue 4 years, will £207·904 purchase, compound interest, £6 per cent.?

First Method.

From 1·06×1·06×1·06×1·06×1·06=1·3382255776
Subt. 1·06×1·06×1·06×1·06 =1·26247696

Divide by $\overline{1\cdot06}^{4}-1$=·26247696)·0757486176(·2885898
·2885898

Multiply by 207·9 present worth.

25973082
20201286
57717960

Ans. 59·99781942=£60

Second Method.

From 1·06×1·06×1·06×1·06×1·06×207·9=278·217097573
Take 1·06×1·06×1·06×1·06×207·9 =262·268959984.

Divide by $\overline{1\cdot06}^{4}-1$=·26247696)15·748137589(
Quotient=59·998=£60.

By TABLE V.

Multiply the tabular number, corresponding with the rate and time, by the purchase money, and the product will be the annuity.

Under £6 per cent. and opposite 4 years, you will find ·28859=annuity which £1 will purchase in 4 years.

Multiply by 207·9

259731
202013
577180

59·997861=£60 Ans.

2. What salary to continue 20 years, will D.688 95c. purchase, at 6 per cent. compound interest? Ans. D.60.

CASE III.

When the annuity, present worth and ratio, are given, to find the time.

RULE.

Divide the annuity by the product of the present worth and ratio subtracted from the sum of the present worth and

annuity, and the quotient will be that power of the ratio, denoted by the number of years, which, being divided by the ratio, and this quotient by the same, till nothing remain, the number of divisions will show the time: Or, the above quotient being sought in Table 1st, under the given rate, in a line with it, you will see the time.

EXAMPLES.

1. For how long may an annuity of £60 per annum be purchased for £207·906336762, at 6 per cent. compound interest?

Multiply 207·906336762
by 1·06

1247438020572
2079063367620

220·38071696772

To 207·906336762=present worth
Add 60· =annuity.

From 267·906336762
Take 220·380716967

47·525619795=divisor.

47·525619795)60·000000000(1·26247696

Divide by 1·06)1·26247696

1·06)1·191016

1·06)1·1236

1·06)1·06

1 } The number of divisions=
time=4 years.

$$\text{Or, } \frac{60}{207\cdot906336762+60-207\cdot906336762\times1\cdot06}=$$

1·26247696, which being sought in Table I. under the given rate, in a line with it, is 4=4 years.

2. How long may a lease of D.300 yearly rent, be had for D.2132·34; allowing 5 per cent. compound interest, to the purchaser? Ans. 9 years.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION AT COMPOUND INTEREST.

When the annuity, time and ratio, are given, to find the present worth of the annuity in reversion.

RULE.

1. Divide the annuity by that power of the ratio denoted by the time of its continuance.

2. Subtract this quotient from the annuity: divide by the ratio less 1, and the quotient will be the present worth, to commence immediately.

3. Divide this quotient by that power of the ratio denoted by the time of reversion, (or, time to come, before the annuity commences) and the quotient will be the present worth of the annuity in reversion.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend.

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it denoted by the time of reversion, and the ratio less 1, continually together for a divisor, and the quotient arising from the division of these two numbers will be the present worth of the annuity in reversion.

EXAMPLES.

1. What is the present worth of £60, payable yearly, for 4 years; but not to commence till two years hence, at £6 per cent.?

First Method.

Ratio=1·06

1·06

636

1060

2d power=1·1236 Carried over.

2d power=1·1236
1·1236

67416
33708
22472
11236
11236

Divide by 4th power=1·26247696)60·000000000000(
Subtract the quotient=47·525619794281

Divide by 1·06—1=·06)12·474380205719

Divide by 1·06×1·06=1·1236)207·9063367619(

Quotient=185·035899=
£185 0s. 8½d.=present worth of the annuity in reversion.

Or, in Table IV. find the present value of £1 at the given rate, both for the time in being and the time in reversion added together, and subtract the present worth of the time in being from the other, multiply the remainder by the annuity, and the product will be the answer.

Present worth of the time in being and reversion=4·91732
Present worth of the time in being=1.8333

3·08402
60

Ans. £185·04120

Or, $\frac{60}{1{\cdot}26247696}=47{\cdot}5256$ $\qquad$ $\frac{60-47{\cdot}5256}{1{\cdot}06-1}=207{\cdot}906$

And $\frac{207{\cdot}906}{1{\cdot}1236}=185{\cdot}035899$

Second Method.

·26247696=4th power—1
Multiply by 60=annuity.

15·74861760=dividend.

1·26247696=4th power.
1·1236=2d power.

757486176
378743088
252495392
126247696
126247696

1·418519112256
·06=ratio—1
·08511114673536=divisor.
·08511115)15·74861760(185·036 Ans.

$$\text{Or,}\ \frac{\overline{1\cdot06}|^4-1\times60}{\overline{1\cdot06}|^4\times\overline{1\cdot06}|^2\times\overline{1\cdot05-1}}=185\cdot036$$

2. What is the present worth of a reversion of a lease of D.60 per annum, to continue 20 years, but not to commence till the end of 8 years, allowing 6 per cent to the purchaser?
Ans. D.431·782 (nearly.)

An annuity, several times in reversion, and rate being given, to find the several present values.

Find the present value of £1 or D.1 by Table 4, at the given rate, and for the several given times, which, being severally multiplied, by the annuity, the products will be the several present values of that annuity, for the several times given: subtract the several present values, the one from the other, and the several remainders will answer the question.

3. A has a term of 6 years in an estate at £60 per annum. B has a term of 14 years in the same estate, in reversion, after the 6 years are expired: and C has a further term of 16 years, after the expiration of 20 years. I demand the present values of the several terms, at 6 per cent.?

	£	s.	d.
Present value of £1 for 36 yrs = 14·61722 × 60 =	877	0	7¼
Ditto of ditto for 20 years = 11·46992 × 60 =	688	3	10¾
Ditto of ditto for 6 years = 4·91732 × 60 =	295	0	9¼

A's term. Therefore, 877 0 7¼ — 688 3 10¾ = £188 16s. 9d. C's term, and 688 3 10¾ — 295 0 9¼ = £393 3 1½ = B's term.

4. For a lease of certain profits for 7 years, A offers to pay D.300 gratuity, and D.300 per annum, B. offers D.800 gratuity and D.250 per annum, C bids D.1300 gratuity and D.200 per annum, and D bids D.2500 for the whole purchase, without any yearly rent; which is the best offer, computing at 6 per cent.?

	D.
By Table IV. the present worth of D.300 per annum, for 7 years, at 6 per cent. is	1674·715
To which add	300·
Value of A's offer =	1974·714
Present worth of D.250 per annum for 7 years =	1395·595
To which add	800·
Value of B's offer =	2195·595
Present worth of D.200 per annum for 7 years =	1116·476
To which add	1300·
Value of C's offer =	2416·476
D's offer =	2500·

Hence it appears that D's offer is the best.

The above questions may be answered by the 4th and 2d Tables.

Take question 1st for example.

1. Multiply the tabular number in Table 4, corresponding to the rate and the time of continuance, into the annuity, and the product will be the present worth, to commence immediately.

2. Multiply this present worth by the tabular number in Table 2, corresponding to the rate and the time of reversion, and the product will be the present worth of the annuity in reversion.

In Table 4th we have 3·4651
Multiply by 60=annuity.

207·9060

In Table 2 we have ·889996

1247436
1871154
1871154
1871154
1663248
1663248

185·035508376=present worth of the reversion.

TABLE FIRST.

Showing the amont of £1 or D.1 from 1 year to forty years.

ys.	5 per cent.	6 per cent.	ys.	5 per cent.	6 per cent.
1	1.0500000	1·0600000	21	2·7859625	3·3995636
2	1·1025000	1·1236000	22	2·9252607	3·6035374
3	1·1576250	1·1910160	23	3·0715237	3·8197496
4	1·2155062	1·2624769	24	3·2250999	4·0489346
5	1·2762815	1·3382256	25	3·3863549	4·2918707
6	1·3400956	1·4185191	26	3·5556726	4·7493829
7	1·4071004	1·5036302	27	3·7334563	4·8223459
8	1·4774554	1·5938480	28	3·9201291	5·1116867
9	1·5513282	1·6894789	29	4·1161356	5·4183879
10	1·6288946	1·7908476	30	4·3219423	5·7434912
11	1·7103393	1·8982985	31	4·5380394	6·0881007
12	1·7958563	2·0121964	32	4·7649414	6·4533867
13	1·8856491	2·1329282	33	5·0031885	6·8405899
14	1·9799316	2·2609039	34	5·2533479	7·2510253
15	2·0789281	2·3965581	35	5·5160152	7·6860868
16	2·1828745	2·5403516	36	5·7918101	8·147252
17	2·2920183	2·6927727	37	6·0814069	8·6860871
18	2·4066192	2·8543391	38	6·3854772	9·1542523
19	2·5269502	3·0255995	39	6·7047511	9·7035074
20	2·6532977	3·2071355	40	7·0399887	10·2857178

TABLE SECOND.

Showing the present value of £1 or D.1 due at the end of any number of years, from 1 to 40.

years.	5 per ct.	6 per ct.	years.	5 per ct.	6 per ct.
1	·952381	·943396	21	·358942	·294155
2	·90703	·889996	22	·34185	·277505
3	·863838	·889619	23	·325571	·261797
4	·822702	·792093	24	·310068	·246978
5	·783526	·747258	25	·305303	·232998
6	·746215	·70496	26	·281241	·21981
7	·710681	·665057	27	·267848	·207368
8	·676839	·627412	28	·255094	·19563
9	·644609	·554898	29	·242946	·184556
10	·613913	·558394	30	·231377	·17411
11	·584679	·562787	31	·220359	·164255
12	·556837	·496969	32	·209866	·154957
13	·530321	·468839	33	·199872	·146186
14	·505068	·442301	34	·190355	·137912
15	·481017	·417265	35	·18129	·130105
16	·458311	·393647	36	·172057	·122741
17	·436297	·371364	37	·164436	·115793
18	·415521	·350343	38	·156605	·109182
19	·395734	·330513	39	·149148	·103002
20	·376889	·311804	40	·142046	·097172

TABLE THIRD.

Showing the amount of £1 or D.1 annuity for any number of years from 1 to 40.

ys.	5 per cent.	6 per cent.	ys.	5 per cent.	6 per cent.
1	1·	1·	21	35·719252	39·992725
2	2·05	2·06	22	38·505214	43·392289
3	3·1525	3·1886	23	41·430475	46·995828
4	4·310125	4·374616	24	44·501999	50·815576
5	5·525631	5·637093	25	47·727099	54·86451
6	6·801913	6·975318	26	51·113454	59·156381
7	8·142008	8·393837	27	54·669126	63·705763
8	9·549109	9·897467	28	58·402583	68·528109
9	11·026564	11·491315	29	62·322712	73·639796
10	12·577892	13·180794	30	66·438847	79·058183
11	14·206787	14·971642	31	70·76079	84·801674
12	15·917126	16·86994	32	75·298829	90·889775
13	17·712983	18·882132	33	80·063771	97.343161
14	19·598632	21·105064	34	85·066959	104·183751
15	21·578563	23·275968	35	90·320307	111·434776
16	23·657492	25·672527	36	95·836323	119·120863
17	25·840366	28·212879	37	101·628139	127·268114
18	28·132385	30·905652	38	107·709546	135·904201
19	30·529004	33·759991	36	114·095025	145·058453
20	33·065954	36·78559	40	120·799774	154·761961

TABLE FOURTH.

Shewing the present value of £1 or D.1 annuity, for any number of years from 1 to 40.

yrs.	5 per ct	6 per ct.	yrs.	5 per ct.	6 per ct.
1	0·95238	0·94339	21	12·82115	11·76407
2	1·85941	1·83339	22	13·163	12·04158
3	2·72325	2·67301	23	13·48807	12·30338
4	3·54595	3·4651	24	13·79864	12·55035
5	4·32948	4·21236	25	14·09394	12·78335
6	5·07569	4·96732	26	14·37518	13·00316
7	5·78637	5·58238	27	14·64303	13·21053
8	6·46321	6·20979	28	14·89813	13·40616
9	7·10782	6·80169	29	15·14107	13·59072
10	7·72173	7·36008	30	15·37245	13·76483
11	8·30641	7·88687	31	15·59281	13·92908
12	8·86325	8·38384	32	15·80268	14·08398
13	9·39357	8·85268	33	16·00255	14·22917
14	9·89864	9·29498	34	16·1929	14·36613
15	10·37966	9·71225	35	16·37419	14·49533
16	10·83777	10·10589	36	16·54685	14.61722
17	11·27407	10·47726	37	16·71129	14·73211
18	11·68958	10·8276	38	16·86789	14·84048
19	12·08532	11·15811	39	17·01704	14·9427
20	12·46221	11·46992	40	17·15909	15·03913

TABLE FIFTH.

The annuity which £1 or D.1 will purchase for any number of years to come, from 1 to 40.

yrs.	5 per ct.	6 per ct.	yrs.	5 per ct.	6 per ct.
1	1·05	1·06	21	·078	·085
2	·5373	·54544	22	·07597	·08303
3	·36721	·37411	23	·07414	·08128
4	·28201	·28859	24	·07247	·07968
5	·23097	·23739	25	·07095	·07823
6	·19702	·20336	26	·06956	·0769
7	·17282	·17913	27	·06829	·0757
8	·15473	·16103	28	·06712	·07459
9	·14069	·14702	29	·06604	·07358
10	·1295	·13587	30	·06505	·07272
11	·12039	·12679	31	·06413	·07179
12	·11282	·11927	32	·06328	·071
13	·10645	·11296	33	·06249	·07027
14	·10102	·10758	34	·06175	·06959
15	·09624	·10296	35	·06107	·06899
16	·09227	·09895	36	·06043	·06839
17	·0887	·09544	37	·05984	·06785
18	·08555	·09235	38	·05928	·06735
19	·08274	·08962	39	·05876	·06689
20	·08024	·08718	40	·05828	·06646

ALLIGATION

IS the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality: It consists of two kinds, viz. Alligation Medial, and Alligation Alternate.

ALLIGATION MEDIAL.

Is, when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

Rule.

As the sum of the quantities, or the whole composition, is to their total value; so is any part of the composition to its mean price or value.

Examples.

1. A Tobacconist would mix 60lb. of tobacco at 6d. per lb. with 50lb. at 1s. 40lb. at 1s. 6d. and 30lb. at 2s. per lb. What is 1lb. of this mixture worth?

	lb.	s.	d.		s.
	60	at 0	6	is	1 10
	50	— 1	0	—	2 10
	40	— 1	6	—	3 0
	30	— 2	0	—	3 0
Sum of the simples	180	Total			10 0

As 180 : 10 :: 1

1

—

10

20

—

180)200(1s.

180

—

20

12

—

180)240(1$\frac{1}{3}$d.

180

—

60

—

Ans. 1s. 1$\frac{1}{3}$d.

2. A farmer would mix 20 bushels of wheat at D.1 per bushel, 16 bushels of rye, at 75c per bushel, 12 bushels of barley, at 50c. per bushel, and 8 bushels of oats at 40c. per bushel: What is the value of one bushel of this mixture?

Ans. 73c. 5$\frac{1}{2}$m.

3. A wine merchant mixes 12 gallons of wine at 75c. per gallon, with 24 gallons, at 90c. and 16 gallons at D.1 10c.: What is a gallon of this composition worth?

Ans. 92c. 0m.

4. A goldsmith melted together 8oz. of gold of 22 carats fine, 1lb. 8oz. of 21 carats fine, and 10oz. of 18 carats fine: Pray what is the quality, or fineness of the composition?

$$\text{Ans. } \frac{8\times22+20\times21+10\times18}{8+20+10.}=20\tfrac{3}{10} \text{ carats fine.}$$

5. A refiner melts 5lb of gold of 20 carats fine with 8lb. of 18 carats fine How much alloy must be put to it to make it 22 carats fine?

$$22-\overline{5\times20+8\times18}\div\overline{5+8}=3\tfrac{3}{13}$$

Answer. It is not fine enough by $3\frac{3}{13}$ carats, so that no alloy must be added, but more gold.

*ALLIGATION ALTERNATE**

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate: so that it is the reverse of Alligation Medial, and may be proved by it.

* By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss upon the whole are equal, and are exactly the proposed rate.

In like manner, let the number of simples be what it may, and with how many soever, each one is linked, since it is always a less with a

CASE I.

The whole work of this case consists in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities sought.

Rule.

1. Place the several prices of the simples, being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase, with a line of connexion at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those, which are greater than the compound, and each greater rate or price with one or any number of the less.

3. Place the difference, between the mean price, or mixture rate, and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

Examples.

1. A merchant has spices, some at 1s. 6d. per lb. some at 2s. some at 4s, and some at 5s. per lb.: How much of each sort must he mix that he may sell the mixture at 3s. 4d. per lb. ?

greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

It is obvious from the rule, that questions of this sort admit of a great variety of answers; for having found one answer, we may find as many more as we please, by only multiplying or dividing each of the quantities found, by 2, 3, 4, &c. the reason of which is evident; for if two quantities of two simples make a balance of loss and gain, with respect to the mean price, so must also the double or triple, the half or third part, or any other ratio of these quantities, and so on *ad infinitum*.

If any one of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

	lb.	lb.	s.	d.	
Mean rate 40d.	18	20	at 1	6	per lb.
	24	8	— 2		
	48	16	— 4		
	60	22	— 5		

	lb.	lb.	s.	d.	
40d.	18	8	at 1	6	per lb.
	24	20	— 2		
	48	22	— 4		
	60	16	— 5		

	d.		lb.	s.	d.	
40d.	18	20+ 8	28	at 1	6	per lb.
	24	8	8	— 2	0	
	48	16+22	38	— 4	0	
	60	22	22	— 5	0	

	d.		lb.	s.	d.	
40d.	18	20	20	at 1	6	per lb.
	24	8+20	28	— 2	0	
	48	16	16	— 4	0	
	60	22+16	38	— 5	0	

	d.		lb.	s.	d.	
40d.	18	8	8	at 1	6	per lb.
	24	8+20	28	— 2	0	
	48	22+16	38	— 4	0	
	60	16	16	— 5	0	

	d.		lb.	s.	d.	
40d.	18	8+20	28	at 1	6	per lb.
	24	20	20	— 2	0	
	48	22	22	— 4	0	
	60	22+16	38	— 5	0	

	d.		lb.	s.	d.	
40d.	18	20+ 8	28	at 1	6	per lb.
	24	8+20	28	— 2	0	
	48	16+22	38	— 4	0	
	60	22+16	38	— 5	0	

NOTE. These seven answers arise from as many vari ways of linking the simples together.

2. *A merchant has Canary wine, at 3s. per gallon, Sherry at 2s. 1d. and Claret at 1s. 5d. per gallon: How much of each sort must he take to sell it at 2s. 4d. per gallon?

Mean rate 28d. { 36 — 3+11 | 14 at 3s. 0d. / 25 — 8 | 8 — 2 1 / 17 — 8 | 8 — 1 5 } per gal.

3. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23, and 24 carats fine, so that the compound may be 21 carats fine: What quantity of each must he take?

Ans. 5oz. of 16 carats fine, 5oz. of 18, 5oz. of 19, 10oz. of 23, and 10 oz. of 24 carats fine.

4. It is required to mix several sorts of wine, at 60c. 90c. and D.1 15c. per gallon, with water, that the mixture may be worth 75c. per gallon: Of how much of each sort must the composition consist?

Ans. 40 gallons of water, 15 galls. of wine at 60c. 15 galls. do. at 90c. and 75 galls. do. at D.1 15.

CASE II.

When the rates *of* all *the* ingredients, *the* quantity *of but* one *of them, and the* mean rate *of the whole mixture are given, to find the several quantities of the rest, in proportion to the quantity given.*

RULE.

Take the difference between each price, and the mean rate, and place them alternately, as in Case I. Then, as the difference standing against that simple, whose quantity is given, is to that quantity, so is each of the other differences, severally, to the several quantities required.

EXAMPLES.

1. A merchant has 40lb. of tea, at 6 shillings per lb. which he would mix with some at 5s. 8d. some at 5s. 2d. and some at 4s. 6d.: How much of each sort must he take to mix with the 40lb. that he may sell the mixture at 5s. 5d. per lb.

* Note, the second question admits but of one way of linking, and so but of one answer; yet all numbers in the same proportion between themselves, as the numbers which compose the answer, will likewise satisfy the condition of the question.

$$65\left\{\begin{array}{ll} 54 & 7+3 \\ 62 & 3+7 \\ 68 & 3+11 \\ 72 & 11+3 \end{array}\right| \begin{array}{l} 10\text{lb.} \\ 10 \\ 14 \\ 14 \text{ stands against the given quantity.} \end{array}$$

$$\text{As } 14 : 40 :: \left\{\begin{array}{l} 10 : 28\frac{8}{14} \text{ at } 4\text{s. } 6\text{d.} \\ 10 : 28\frac{8}{14} - 5\ \ 2 \\ 14 : 40 - 5\ \ 8 \end{array}\right\} \text{per lb.}$$

2. How much gold of 16, 20, and 24 carats fine, and how much alloy, must be mixed with 10 oz. of 18 carats fine, that the compesition may be 22 carats fine.

Ans. 10 oz. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

Alternation Total.

CASE III.

When the rates *of the several ingredients, the* quantity *to be compounded, and the* mean rate *of the whole mixture are given, to find how much of each sort will make up the quantity.*

Rule.

Place the differences between the mean rate, and the several prices alternately, as in Case I.; then, as the sum of the quantities, or differences, thus determined, is to the given quantity, or whole composition: so is the difference of each rate, to the required quantity of each rate.

Examples.

1. Suppose I have four sorts of currants, at 8d. 12d. 18d. and 22d per lb; the worst will not sell, and the best are too dear; I therefore conclude to mix 120lb. and so much of each sort as to sell them at 16d. per lb.: How much of each sort must I take?

$$16\text{d.}\left\{\begin{array}{ll} \text{d.} & \text{lb.} \\ 8 & 6 \\ 12 & 2 \\ 18 & 4 \\ 22 & 8 \end{array}\right. \qquad \text{Sum}=20$$

$$\text{As } \begin{array}{ll} \text{lb.} & \text{lb.} \\ 20 & 120 \end{array} :: \left\{\begin{array}{lll} \text{lb.} & & \text{d.} \\ 6 : 36 & \text{at} & 8 \\ 2 : 12 & - & 12 \\ 4 : 24 & - & 18 \\ 8 : 48 & - & 22 \end{array}\right\} \text{per lb.} \qquad 120$$

2. A goldsmith has several sorts of gold; viz. of 16, 17, 20 and 22 carats fine, and would melt together, of all

these sorts, so much as may make a mass of 40oz. 18 carats fine; how much of each sort is required?

Ans. 16oz. 15 carats fine, 8oz. 17, 4 oz. 20, and 12oz. of 22 carats fine.

3. A merchant would mix four sorts of wine, of several prices; viz. at 75c. D.1 25c. D.1 50c. and D.1 62½c. per gallon; of these he would have a mixture of 72 gallons, worth D.1 37½c. per gallon; what quantity of each sort must he have.

Ans. 8 at 75c. 16 at D.1 25c. 40 at D.1 50c. and 8 at D.1 62½ c. Or, 16 at 75c. 8 at D.1 25c. 8 at D.1 50c. and 40 at D.1 62½c.

CASE IV.

When more than one of the Simples are limited.

RULE.

Find, by Alligation Medial, what will be the *rate* of the mixture made of the *given quantities* of the *limited simples* only; then, consider this as the *rate* of a *limited simple*, whose *quantity* is the sum of the first given *limited simples*, from which, and the *rates* of the *unlimited simples*, by Case 2d. calculate the quantity.

EXAMPLES.

1. How much wine at 80c. and at 87½c. per gallon, must be mixed with 8 gallons at 75c. and 12 gallons at 90c. per gallon, that the mixture may be worth 82½c. per gallon?

Limited simples { 8 gallons, at 75c. = D. 6 / 12 gallons, at 90 = 10 80c. }

20 — 16 80

Gal. D. c. Gal. c.

As 20 : 16 80 :: 1 : 84 per gallon.

Now, having found the rate of the limited simples, the question may stand thus: How much wine, at 80c. and 87½c. per gallon must be mixed with 20 gallons at 84c. per gallon, that the mixture may be worth 82½c. per gallon?

$82\frac{1}{2}$ { 80, 84, $87\frac{1}{2}$ } $1\frac{1}{2}+5$ | $2\frac{1}{2}$ | $2\frac{1}{2}$ | $6\frac{1}{2}$ gallons at 80c. | 84 | $87\frac{1}{2}$

As $2\frac{1}{4}$: $\left\{\begin{matrix}6\frac{1}{2}\\2\frac{1}{4}\end{matrix}\right\}$:: 20 : $\left\{\begin{matrix}52\text{ galls. at }80\text{c. per gall.}\\20\text{ ——— }87\frac{1}{2}\text{c. ———}\end{matrix}\right\}$ Ans.

Proof.

52	gallons at	80c.	=	D.41	60c.
50	———	$87\frac{1}{2}$	=	17	50
8	———	75	=	6	
12	———	90	=	10	80
92	———	$82\frac{1}{2}$	=	75	90

POSITION.

POSITION is a rule, which, by false or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts; *single* and *double*.

SINGLE POSITION.

Single Position teaches to resolve those questions, whose results are proportioned to their suppositions; such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times.

Rule.

1. Take any number, and perform the same operations with it as are described to be performed in the question.

2. Then say, as the sum of the errors is to the given sum, so is the supposed number to the true one required.

Proof. Add the several parts of the sum together, and if it agrees with the sum, it is right.

Examples.

1. A school-master, being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth *as many*, I should then have 435: Of what number did his *school consist?*

Suppose he had 80	As 290 : 435 :: 80	
As many=80	80	120
¾ as many=60	——	120
½ as many=40	29\|0)3480\|0(120 Ans.	90
¼ as many=20	29	60
⅛ as many=10	—	30
——	58	15
290	58	——
	—	Proof 435
	0	

2. A person lent his friend a sum of money unknown, to receive interest for the same at 6 per cent. per annum, simple interest, and at the end of 12 years, received for principal and interest D.860: What was the sum lent?

Ans. D.500.

3. A, B and C joined their stocks, and gained D.353 12½c. of which A took up a certain sum, B took up four times so much as A, and C, five times so much as B: What share of the gain had each?

Ans. D. 14 12½c. A's share.
56 50 B's share.
282 50 C's share.

4. A, B, C and D spent 35s. at a reckoning, and being a little dipped they agreed that A should pay ⅓, B ½, C ⅕, D ¼: What did each pay in the above proportion?

Ans. A 13s. 4d.
B 10 0
C 6 8
D 5 0

5. A certain sum of money is to be divided betweeen 5 men, in such a manner as that A shall have ¼, B ½, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is £40: What is the sum?

Suppose £200. Then $\frac{1}{4}+\frac{1}{2}+\frac{1}{10}+\frac{1}{20}=120$.

200—120=80. As 80 : 40 :: 200 : 100 Ans.

6. A and B, talking of their ages, B said his age was once and an half the age of A; C said his was twice and one tenth the age of both, and that the sum of their ages was 93: What was the age of each?

Ans. A's 12, B's 18, and C's 63 years.

7. A and B having found a purse of money, disputed who should have it: A said that $\frac{1}{3}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to £35, and if B could tell him how much was in it he should have the whole, otherwise he should have nothing: How much did the purse contain?

Ans. £100.

8. A gentleman divided his fortune among his sons; to A he gave D.9 as often as to B D.5, and to C but D.3 as often as to B D.7, yet C's portion came to D.1059: What was the whole estate? Ans. D.7977 80c.

9. Seven eights of a certain number exceeds four fifths by 6: What is that number? Ans. 80.

DOUBLE POSITION.

Double Position teaches how to resolve questions by making two suppositions of false numbers.

Those questions, in which the results are not proportional to their positions, belong to this rule: such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

RULE.*

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the results or errors against their positions or supposed numbers, thus,

Pos.	Err.
30	12
20	6

and if the error be too great, mark it with +; and if too small with —.

3. Multiply them crosswise; that is, the first position by the last error, and the last position by the first error.

4. If the errors be alike, that is, both too small or both too great, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

* The rule is founded on this supposition, that the first error is to the second as the difference between the true and first supposed number is to the difference between the true and second supposed number: When that is not the case, the exact answer to the *question cannot* be found by this rule.

5. If the errors be unlike, that is, one too small, and the other too great, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Note. When the errors are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

1. A lady bought damask for a gown, at 8s. per yard, and lining for it at 3s. per yard; the gown and lining contained 15 yards, and the price of the whole was £3 10s.: How many yards were there of each?

Suppose 6 yards damask, value 48s.
Then she must have 9 yards lining, value 27s.

Sum of their values=75s.
So that the first error is 5 too much, or+ 5
Again, suppose she had 4 yards of damask, value 32s.
Then she must have 11 yards of lining, value 33s.

Sum of their values=65s.
So that the second error is 5 too little, or — 5

Then $\begin{matrix}6 & 5+\\ 4 & 5-\end{matrix}$ X

5 yards at 8s=	£2	0	6
10yds. at 3s.=	1	10	0
Proof	3	10	0

20 30
20

Sum of errors=5+5=10)50

Ans. 5yds. damask, and 15—5=10 yards lining. Or 6+4÷2=5 as before.

2. A labourer was hired for 60 days, upon this condition, that, for every day he wrought, he should receive 75c.; and for every day he was idle, should forfeit 37½c.; at the expiration of the time he received D.18: How many days did he work, and how many was he idle?

Suppose he worked $\begin{cases}20 & 18-\\ 40 & 4\frac{1}{2}+\end{cases}$ X

Ans. He was employed 36 days, and was idle 24.

3. A gentleman has two horses of considerable value, and a carriage worth £100; now if the first horse be harnessed in it, he and the carriage together will be triple the value of the second; but if the second be put in they will be 7 times the value of the first: What is the value of each horse?

Suppose $\begin{cases} 32 & 80- \\ 44 & 160- \end{cases}$ Ans. One £20, the other £40.

4. What number is that, which being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be doubled?

Suppose $\begin{cases} 8 & 3+ \\ 16 & 1+ \end{cases}$ Ans. 20.

5. A farmer having driven his cattle to market, received for them all D.320, being paid at the rate of D.24 per ox, D.16 per cow, and D.6 per calf; there were as many oxen as cows, and 4 times as many calves as cows: How many were there of each sort?

Suppose cows $\begin{cases} 6 & 64+ \\ 12 & 448+ \end{cases}$ Ans. 5 oxen, 5 cows, and 20 calves.

PERMUTATIONS and COMBINATIONS.

The Permutation of Quantities is the shewing how many different ways any given number of things may be changed.

This is also called *variation alternation* or *changes;* and the only thing to be regarded here is the order they stand in; for no two parcels are to have all their quantities placed in the same situation.

The combination of quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

This is sometimes called *election* or *choice;* and here every parcel must be different from all the rest, and no two *are to have precisely* the same quantities, or things.

The Composition of Quantities is the taking of a given number of quantities, out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places; and it differs from Combinations only as that admits but of one row of things.

Problem I.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

Rule.*

Multiply all the terms of the natural series of numbers, from 1 up to the given numbers, continually together, and the last product will be the answer required.

Examples.

1. Christ church, in Boston, has 8 bells: how many changes may be rung on them?

$1\times2\times3\times4\times5\times6\times7\times8=40320$ Ans.

2. Nine gentlemen met at an inn, and were so pleased with their host, and with each other, that in a frolick they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner. Pray how long, had they kept their agreement, would their frolick have lasted? Ans. $9941\frac{33}{80}$ years.

3. How many changes, or variations, will the alphabet admit of? Ans. 620448401733239439360000.

Problem II.

Any number of different things being given, to find how many changes can be made out of them by taking any given number of quantities at a time.

Rule.

Take a series of numbers, beginning at the number of things given, and decreasing by 1, as many terms as the

* Any two things a and b are capable of two variations only; as ab, ba; whose number is expressed by 1×2.

If there be three things, a, b, and c; then any two of them, leaving out the third, will have 1×2 variations; and consequently when the third is taken in, there will be $1\times2\times3$ variations, and so on, as far as you please.

number of quantities to be taken at a time; the product of all the terms will be the answer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

8
7
—
56
6
—
336
5
—
1680

Or, $8\times7\times6\times5$ (=4 terms)=1680 Ans.

2. How many words can be made with 6 letters of the alphabet, admitting a number of consonants may make a word?

$24\times23\times22\times21\times20\times19$ (6 terms)=96909120, Ans.

PROBLEM III.

Any number of things being given, whereof there are several things of one sort, several of another, &c. to find how many changes may be made out of them all.

RULE.*

1. Take the series $1\times2\times3\times4$, &c. up to the number of things given, and find the product of all the terms.

* Any 2 quantities, *a*, *b*, both different, admit of 2 changes; but if the quantities are the same, or *ab* becomes *aa* there will be only one alteration, which may be expressed by $\frac{1\times2}{1\times2}=1$.

Any 3 quantities, *a*, *b*, *c*, all different from each other, admit of 6 variations; but if the quantities are all alike, or *abc* become *aaa*, then the 6 variations will be reduced to 1, which may be expressed by $\frac{1\times2\times3}{1\times2\times3}=1$. Again, if 2 quantities out of 3 are alike, or *abc* become *aac*; then the 6 variations will be reduced to these 3, *aac caa*, *aca*, which may be expressed by $\frac{1\times2\times3}{1\times2}=3$, and so of any greater number.

2. Take the series 1×2×3×4, &c. up to the number of the given things of the first sort, and the series, 1×2×3×4, &c. up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series by the *joint* product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zapknathpaaneah?*

1×2×3×4×5×6×7×8×9×10×11×12×13×14×15 (=number of letters in the word) =1307674368000.

1×2×3×4×5 (=number of *as*) =120
1×2 (=number of *ps*) = 2
1 (=number of *ts*) = 1
1×2×3 (=number of *hs*) = 6
1×2 (=number of *ns*) = 2

2×6×1×2×120=2880)1307674368000(454053600 Ans.

2. How many different numbers can be made of the following figures, 1223334444? Ans. 12600.

PROBLEM IV.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.*

1. Take the series 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1 from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

* In any given number of quantities, the number of Combinations increases gradually till you come about the mean numbers, and then gradually decreases. If the number of quantities be *even*, half the number of places will shew the greatest number of Combinations that can be made of those quantities; but if odd, then those two numbers which are the middle, and whose sum is equal to the given number of quantities, will shew the greatest number of Combinations.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12?

1×2×3×4×5×6×7 (=the number to be taken at a time) =5040.

12×11×10×9×8×7×6 (=same number from 12)=3991680.

5040)3991680(792 Ans.

2. How many combinations can be made, of 6 letters out of the 24 letters of the alphabet?

Ans. 134596.

3. A farmer bargained with a gentleman for a dozen sheep, (at 2 dollars per head) which were to be picked out of 2 dozen; but being long choosing them, the gentleman told him that if he would give him a cent for every different dozen which might be chosen out of the two dozen, he should have the whole, to which the farmer readily agreed: Pray what did they cost him?

Ans. D.27041 56c.

PROBLEM V.

To find the compositions of any number, in an equal number of sets, the things being all different.

RULE.

Multiply the number of things in every set continually together, and the product will be the answer required.

EXAMPLES.

1. Suppose there are 5 companies, each consisting of 9 men; it is required to find how many ways 5 men may be chosen, one out of each company?

Multiply 9 into itself continually, as many times as there are companies.

9×9×9×9×9=59049 different ways, Ans.

2. How many changes are there in throwing 4 dice?

As a die has 6 sides, multiply 6 into itself four times continually. 6×6×6×6=1296 changes, Ans.

3. In how many ways may a man, a woman, and a child be chosen out of three companies, consisting of 5 men, 7 women and 9 children? Ans. 315.

THE USE OF LOGARITHMS.

1. In Multiplication.

Given two numbers, viz. 275 and 12·6 to find their product.

Rule, To the logarithm of 275, viz.	2·43933
Add the logarithm, of 12·6, viz.	1·10037
And their sum is the logarithm of their product, viz.	3465=3.53970

2. In Division.

Let it be required to find the quotient, which arises by dividing one number by another; suppose 1425 by 57.

From the logarithm of the dividend, viz.	1425=3·15381
Take the logarithm of the divisor, viz. 57	=1·75587
And the remainder is the logarithm of the quotient, viz.	25=1·39794

3. In the Rule of Three.

Three numbers given to find a fourth, in direct proportion.

Rule. From the tables take the logarithms of each of the proposed numbers, then add the logarithms of the second and third together, and from the sum, take the logarithm of the first, and the remainder will be the logarithm of the o urth number.

Let the three proposed numbers be 18, 24 and 33, and the operation will stand thus:

1·38021=the logarithm of 24, the 2d term.
1·51851=the logarithm of 33, the 3d term.

2·89872=the logarithm of their product.
—1·25527=the logarithm of the 1st term 18.

1·64345=the logarithm of the fourth term required, which, by the table, answers to the natural number 44, the 4th proportional to the three proposed numbers.

4. In Involution, or Raising Powers.

To find any power of any proposed number, or to involve any number to any proposed power, by logarithms.

Rule. Multiply the logarithm of the given root by the power, viz. by 2 for the square, by 3 for the cube, &c. and the product is the logarithm of the power sought.

Required to find the cube of 12?

1·07918=the logarithm of 12.
×3=the third power, or cube.

3·23754=1728 the cube of 12.

5. In Evolution, or Extracting Roots.

To extract any root of any proposed number.

Rule. Divide the logarithm of the proposed number by the index of the required root, viz. by 2 for the square, by 3 for the cube, &c. and the quotient will be the logarithm of the root required.

Required to find the cube root of 1728?

3·23754=the logarithm of 1728, and 3·23754÷3= 1·07918 is the logarithm of the cube root of 1728, viz. 12.

6. In Compound Interest.

To find the amount of any sum for any time and at any rate, at compound interest.

Rule. Multiply the logarithm of the ratio (i. e. the amount of £1 or D.1 for 1 year) by the number of years, and to the product add the logarithm of the principal; the sum will be the logarithm of the amount.

What will £45 amount to, forborne 12 years, at 6 per cent. per annum, compound interest?

Log. of 1·06, the ratio is ·02533
Multiply by the time 12

·30396
Add Log. of 45 the principal 1·65321

The sum is 1·95717 which is the logarithm of 90·7=£90 14s. Ans.

7. In Discount at Compound Interest.

To find the present worth of any sum of money, due any time hence, at any rate, at compound interest

Rule. From the logarithm of the sum to be discounted, subtract the logarithm of the rate multiplied by the time; and the remainder is the logarithm of the present worth.

What present money will pay a debt of £90 14s. due 12 years hence, discounting at the rate of 6 per cent. per annum?

From the logarithm of £90 14 = 1·95717

Sub. product of the Log. of the ratio × by the time, = ·30396

The remainder 1·65321 is the logarithm of £45, Ans.

RULES

For reducing Federal Money to the several former currencies of the United States, and to English Money, and the contrary.

	s.	d.	
D.1 =	6	0	old currency of New-England and Virginia.
	8	0	New-York and N. Carolina.
	7	6	New-Jersey, Pennsylvania, Delaware and Maryland.
	4	8	South Carolina and Georgia.
	4	6	Sterling or English Money

Note. Most of the operations under the following rules are by decimals; so that, if pounds with lower denominations be given, the lower denominations must be reduced to decimals of a pound; and, if dollars, &c. be reduced to pounds and decimals, the decimals may be reduced to shillings, &c. in the answer.

I. *To reduce Federal Money and the old currency of New-England and Virginia to each other.*

Multiply dollars by ·3, or divide by $3\frac{1}{3}$ for pounds.
Multiply pounds by $3\frac{1}{3}$, or divide by ·3 for dollars.

EXAMPLES.

1. Reduce D.150 to pounds.

·3

£45·0 Ans.

Or, $3\frac{1}{3}=\frac{10}{3}$; and,
$150\div\frac{10}{3}=150\times\frac{3}{10}=\frac{450}{10}=$
£45 Ans. as before.

2. Reduce £45 to dollars.

$3\frac{1}{3}$

135
15

D.150 Ans.

Or, ·3)45·

D.150· Ans. as before.

3. Reduce D.17 81c, $2\frac{1}{2}$m. to pounds, &c.

17·8125
·3

£5·34375
20

s. 6·875
12

d. 10·5
4

qr. 2·0

Ans. £5 6s. $10\frac{1}{2}$d.

Or, $3\frac{1}{3}=\frac{10}{3}$; and 17·8125 $\div\frac{10}{3}=17\cdot8125\times\frac{3}{10}$, or ×·3; hence the operation and result as before,

4. Reduce £5 6s. $10\frac{1}{2}$d. to dollars, &c.

4	2· qrs.
12	10·5 d.
20	6·875 s.

£5·34375
$3\frac{1}{3}$

16·03125
1·78125

D.17·8125 Ans.

Or, ·3)5·34375

D.17·8125 Ans. as before.

5. Reduce D.271 17c. 5m. to pounds, &c.

Ans. £81 7s. $0\frac{1}{2}$d.+

6. Reduce £81 7s. $0\frac{1}{2}$d. to dollars, &c.

Ans. D.271 17c. 5m.—

7. Reduce D.138 62c. 5m. to pounds, &c.
Ans. £41 11s. 9d.

8. Reduce £41 11s. 9d. to dollars, &c.
Ans. D.138 62c. 5m.

II. *To reduce Federal Money to the old currency of New York and North-Carolina, and the contrary.*

Multiply dollars by ·4, or divide by 2·5, for pounds.
Multiply pounds by 2·5, or divide by ·4 for dollars.

EXAMPLES.

1. Reduce 59 dollars to pounds, &c.

59
·4
£23·6
20
s. 12·

Ans. £23 12s.

Or, 2·5)59·0(23 £ s. 12 Ans.
50 as before.
90
75
15
20
300
25
50
50

2. Reduce £23 12s. to dollars, &c.

£23 12s.=£23·6
2·5
1180
472
D.59·00 Ans.

Or, ·4)23·6
D.59· Ans. as before.

3. Reduce D.212 43c. 7½m. to pounds, &c.

D.212·4375×·4=£84·975
=£84 19s. 6d. Ans.

Or, 212·4375÷2·5=
£84·975=£84 19s. 6d.
Ans. as before.

4. Reduce £84 19s. 6d. to dollars, &c.

£84 19s. 6d.=£84·97
£84·975×2·5=D.212·437
Ans.

Or, £84·975÷·4=
D.212·4375 Ans. as before

5. Reduce D.74 33⅓c. to pounds, &c.
Ans. £29 14s. 8d.

6. Reduce £29 14s. 8d. to dollars, &c.
Ans. D.74 33⅓c.

III. *To reduce Federal Money and the old currency of New-Jersey, Pennsylvania, &c. to each other.*

Multiply dollars by ·375, or divide by 2⅔, for pounds.
Multiply pounds by 2⅔, or divide by ·375 for dollars

EXAMPLES.

1. Reduce D.59 75c. to pounds, &c.

D.59·75
·375
29875
41825
17925
£22·40625=
£22 8s. 1½d. Ans.

Or, 2⅔=8/3; and D.59·75 ÷ 8/3 = 59·75 × 3/8 = 179·25/8 =
£22·40625 as before.

2. Reduce £22 8s. 1½d. to dollars, &c.

£22 8s. 1½d.=£22·40625
2⅔
44·81250
14·9375
Ans. D.59·75

Or,
·375)22·40625(D.59·75
Ans. as before.

3. Reduce D.79 87c. 5m. to pounds, &c.
Ans. £30 19s. 0¾d.

4. Reduce £30 19s 0¾d. to dollars, &c.
Ans. D.79 87c. 5m.

Reduce D.131 80c. to pounds, &c.
Ans. £49 8s. 6d.

6. Reduce £49 8s. 6d. to dollars, &c.
Ans. D.131 80c.

IV. *To reduce Federal Money and the old currency of South-Carolina and Georgia to each other.*

Multiply dollars by ·2⅓, or divide by 4⅔, for pounds.
Multiply pounds by 4⅔, or divide by ·2⅓, for dollars.

EXAMPLES.

1. Reduce 375 dollars to pounds, &c.

375·
·2½

75·0
12·5

£87·5
20

s. 10.

Or, $4\frac{2}{7}=\frac{30}{7}$; and $375\div\frac{30}{7}=375\times\frac{7}{30}=\frac{2625}{30}=87\cdot5$ as before.

2. Reduce £87 10s. to dollars, &c.

£87 10s. = £87.5
4

350·
12·5=⅛
12·5=⅛

D.375· Ans.

Or, $\cdot2\frac{1}{2}=\frac{\cdot7}{3}$; and $\pounds87\cdot5\div\frac{\cdot7}{3}=\pounds87\cdot5\times\frac{3}{\cdot7}=\frac{262\cdot5}{\cdot7}$ =D.375· Ans. as before.

3. Reduce D.67 87c. 5m. to pounds, &c. Ans. £15 16s. 9d.

4. Reduce £15 16s. 9d. to dollars, &c. Ans. D.67 87c. 5m.

5. Reduce D.100 to pounds, &c. Ans. £23 6s. 8d.

6. Reduce £23 6s. 8d. to dollars, &c. Ans. D.100.

V. *To reduce Federal Money to English Money, and the contrary.*

Multiply dollars by ·225, or divide by $4\frac{4}{9}$ for pounds.
Multiply pounds by $4\frac{4}{9}$, or divide by ·225, for dollars.

EXAMPLES.

1. Reduce D.444 $44\frac{4}{9}$c. to pounds.

D.444 $44\frac{4}{9}$
·225

222220
88888
88888
100=⅛ of ·225

£100·00000 Ans.

Or, $4\frac{4}{9}=\frac{40}{9}$; and $444{\cdot}44\frac{4}{9}=\frac{4000}{9}$; then, $\frac{4000}{9}\div\frac{40}{9}=\frac{4000}{9}\times\frac{9}{40}=\frac{36000}{360}=$£100 Ans. as before.

2. Reduce £100 to dollars, &c.

$$\begin{array}{r} 4\frac{4}{9} \\ \hline 400 \\ \hline 11{\cdot}11\frac{1}{9}=\frac{1}{9} \\ \hline 44{\cdot}44\frac{4}{9}=\frac{4}{9} \end{array}$$

D.444·44$\frac{4}{9}$ Ans.

Or, ·225)100·(D.444·44$\frac{4}{9}$ Ans. as before.

3. Reduce D.25 to pounds, &c.

25×·225=5·625; and £5·625=£5 12s. 6d. Ans.

Or, $\frac{25}{4\frac{4}{9}}$=5·625, as before.

4. Reduce £5 12s. 6d. to dollars.

£5 12s. 6d.=£5·625; and 5·625×$4\frac{4}{9}$=D.25 Ans.

Or, ·225)5·625(D.25 Ans. as before.

5. Reduce D.36 93c. 7$\frac{1}{2}$m. to pounds, &c.

Ans. £8 6s. 2d. 2$\frac{1}{2}$qr.

6. Reduce £8 6s. 2d. 2$\frac{1}{2}$qr. to dollars, &c.

Ans. D.36 93c. 7$\frac{1}{2}$m.

USEFUL TABLES.

A TABLE, directing how to buy and sell by the hun. Weight, in Federal Money.

D. D.d. d.c. c.m.	D. D.d. D.d.c. D.d.c.m.	D. D.d. d.c. c.m.	D. D.d. D.d c. D.d.cm.	D. D.d. d.c. c.m.	D. D.d. D.d.c. D.d.c.m
1	112	34	3808	67	7504
2	224	35	3920	68	7616
3	336	36	4032	69	7728
4	448	37	4144	70	7840
5	560	38	4256	71	7952
6	672	39	4368	72	8064
7	784	40	4480	73	8176
8	896	41	4592	74	8288
9	1008	42	4704	75	8400
10	1120	43	4816	76	8512
11	1232	44	4928	77	8624
12	1344	45	5040	78	8736
13	1456	46	5152	79	8848
14	1568	47	5264	80	8960
15	1680	48	5376	81	9072
16	1792	49	5488	82	9184
17	1904	50	5600	83	9296
18	2016	51	5712	84	9408
19	2128	52	5824	85	9520
20	2240	53	5936	86	9632
21	2352	54	6048	87	9744
22	2464	55	6160	88	9856
23	2576	56	6272	89	9968
24	2688	57	6384	90	10080
25	2800	58	6496	91	10192
26	2912	59	6608	92	10204
27	3024	60	6720	93	10316
28	3136	61	6832	94	10428
29	3248	62	6944	95	10540
30	3360	63	7056	96	10652
31	3472	64	7168	97	10764
32	3584	65	7280	98	10876
33	3696	66	7392	99	10988

A TABLE for reducing Troy Weight to Avoirdupois.

Troy.	Avoir	Troy	Avoirdupois.		
gr.	drams	oz.	lb.	oz.	drams.
1	·04	1		1	1 55
2	·07	2		2	3·11
3	·11	3		3	4·66
4	·15	4		4	6·22
5	·18	5		5	7·77
6	·22	6		6	9·33
7	·26	7		7	10·89
8	·29	8		8	12·44
9	·33	9		9	14·
10	·36	10		10	15 56
11	·4	11		11	1·09
12	·44	lb.			
13	·47	1	0	13	2·65
14	·51	2	1	10	5·3
15	·55	3	2	7	8·
16	·58	4	3	4	10 6
17	·62	5	4	1	13·25
18	·66	6	4	14	15·9
19	·69	7	5	12	2·56
20	·73	8	6	9	5 21
21	·77	9	7	6	7·86
22	·8	10	8	3	10·52
23	·84	20	16	7	5·03
pwt.		30	24	10	15·54
1	0·88	40	32	14	10 05
2	1·75	50	41	2	4·57
3	2·63	60	49	5	15·08
4	3·51	70	57	9	9 6
5	4·39	80	65	13	4·11
6	5·27	90	74	0	13·62
7	6·14	100	82	4	9·15
8	7·02	200	164	9	2·28
9	7 9	300	246	13	11·42
10	8·78	400	329	2	4·57
11	9·65	500	411	6	13·71
12	10 53	600	493	11	6 85
13	11·41	700	576	0	0·
14	12·29	800	658	4	9·14
15	13 16	900	740	9	2·28
16	14·04	1000	822	13	11·42
17	14·92	2000	1645	11	6·84
18	15·79	3000	2528	9	2·26
19	16·67	4000	3291	6	13·68

A TABLE for reducing Avoirdupois Weight to Troy Weight.

Av.	*Troy.*				*Avoir*	*Troy*			
dr.	lb.	*oz.*	*pwt.*	*gr.*	lb.	lb.	*oz.*	*pwt.*	*gr.*
½				13·67	1	1	2	11	16
¾				20·51	2	2	5	3	8
1			1	3·34	3	3	7	15	0
2			2	6·68	4	4	10	6	16
3			3	10·02	5	6	0	18	8
4			4	13·36	6	7	3	10	0
5			5	16·7	7	8	6	1	16
6			6	20·04	8	9	8	13	8
7			7	23·38	9	10	11	5	0
8			9	2·72	10	12	1	16	16
9			10	6·06	20	24	3	13	8
10			11	9·4	30	36	5	10	0
11			12	12·74	40	48	7	6	16
12			13	15·08	50	60	9	3	8
13			14	19·42	60	72	11	0	0
14			15	22·76	70	85	0	16	16
15			17	2·1	80	97	2	13	8
oz.					90	109	4	10	0
1			18	5·5	100	121	6	6	16
2		1	16	11	200	243	0	13	8
3		2	14	16·5	300	364	7	0	0
4		3	12	22	400	486	1	6	16
5		4	11	3·5	500	607	7	13	8
6		5	9	9	600	729	2	0	0
7		6	7	14·5	700	850	8	6	16
8		7	5	20	800	972	2	13	8
9		8	4	1·5	900	1093	9	0	0
10		9	2	7	1000	1215	3	6	16
11		10	0	12·5	2000	2430	6	13	8
12		10	13	18	3000	3645	10	0	0
13		11	16	23·5	4000	4861	1	6	16
14	1	0	15	5	5000	6076	4	13	8
15	1	1	13	10·5	6000	7291	8	0	0

By a law of the Congress of the United States, foreign gold and silver coins are made a legal tender for the payment of all debts and demands, at the several and respective rates following, viz. The gold coins of Great-Britain and Portugal, of their present standard, at the rate of 100 cents for every 27 grains of the actual weight thereof—Those of France and Spain 27⅛ grains of the actual weight thereof—Spanish milled dollars, weighing 17 pwt. 7grs. equal to 100 cents, and in proportion for the parts of a dollar—Crowns of France, weighing 18pwt. 17grs. equal to 110 cents, and in proportion for the parts of a crown. It is also enacted, That every cent shall contain 208 grains of copper, and every half cent 104 grains; agreeably to which the following Tables are calculated.

A TABLE of the Weights of several pieces of English, Portuguese, and French Gold Coins, according to an act of Congress, passed Nov. 1792.

	Pwt.	Gr.	Dols.	Cents.	Mills.
Double Johannas	18		16	0	0
Single ditto	9		8	0	0
English Guinea	5	6	4	66⅔	0
Half ditto	2	15	2	33⅓	0
French Guinea	5	6	4	59	8
Half ditto	2	15	2	29	9
4 Pistoles	16	12	14	45	2
2 Pistoles	8	6	7	22	6
1 Pistole	4	3	3	61	3
Moidore	6	22	6	14	8

A TABLE of the value of English and Portuguese Gold, in Dollars, Cents, and Mills, throughout the United States.

Gr.	Cts.	m.	Pwt.	Dols.	Cts.
1	3	7	1	0	89
2	7	4	2	1	77¾
3	11	1	3	2	66⅔
4	14	8	4	3	55½
5	18½	0	5	4	44
6	22	2	6	5	33⅓
7	25	9	7	6	22
8	29	6	8	7	11
9	33⅓	0	9	8	0
10	37	0	10	8	89
11	40	7	11	9	77¾
12	44	4	12	10	66⅔
13	48	1	13	11	55½
14	51	8½	14	12	44½
15	55	5⅓	15	13	33⅓
16	59¼	0	16	14	22
17	63	0	17	15	11
18	66⅔	0	18	16	0
19	70	4	19	16	89
20	74	0	1 oz.	17	77¾
21	77¾	0			
22	81½	0			
23	85	2			

NOTE. 89 cents is the value of 1 pennyweight of English and Portuguese gold.

A TABLE of the value of French and Spanish Gold, in Dollars, Cents, and Mills, throughout the United States.

Gr.	Cts.	m.	Pwt.	Dols.	Cts.	m.
1	3	6	1	0	87	6
2	7	3	2	1	75	2
3	11	0	3	2	62½	0
4	14	6	4	3	50	3
5	18	2	5	4	38	0
6	21	9	6	5	25	5
7	25	5	7	6	13	1
8	29	2	7	7	0	7
9	32	8	8	8	88	3
10	36⅓	0	9	9	76	0
11	40	1	10	10	63½	0
12	43	8	11	11	51	1
13	47	4	12	12	38	7
14	51	1	13	13	26	3
15	54¾	0	14	14	13	9
16	58	4	14	15	1½	0
17	62	0	15	16	89	0
18	65	7	16	17	76	6
19	69	3	17	18	64	2
20	73	0	1 oz	19	51	8
21	76	6				
22	80	3				
23	83	9				

NOTE. 87 cents 6 mills is the value of 1 pennyweight French and Spanish Gold.

FINIS.

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